

Inertial Oscillations, Entrainment, and Critical Length Scales of Falling Spheres in Stratified Fluids

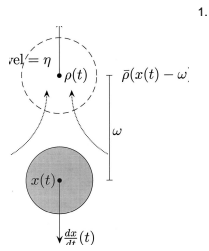
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$$\text{Eq. 1 } m \frac{d^2 X}{dt^2} - F \left(2 \frac{dX}{dt} - \eta(t) \right) = mg \left(\frac{p_b - \bar{p}(X(t))}{p_b} \right)^{1.}$$

$$\text{Eq. 2 } z \frac{d\eta(t)}{dt} - F \left(\eta(t) - \frac{dX}{dt} \right) = z g \left(\frac{\bar{p}(X(t) - \omega) - p(t)}{p(t)} \right)$$

These two coupled ODEs were used to model the behavior of the sphere in the 2004 paper "An internal splash: Levitation of falling spheres in stratified fluids." It was discovered that the transverse motion of the sphere sometimes experienced could be explained by considering the system to be composed of two spheres, one real sphere with the same physical parameters as the one used in the experiment and one imaginary one composed of the lower density fluid that comprises the top layer. Both equations simply state that the buoyancy force of each sphere is equal to the difference gravitational acceleration and a frictional force caused by the surrounding fluid; Eq. 1 applied to the real sphere and Eq. 2 applies to the imaginary one.

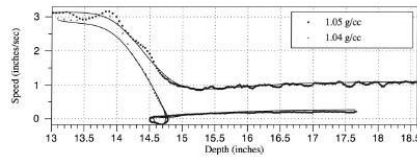


1. Schematic depicting model parameters: Dynamic fluid replaced by idealized fluid particle, located a fixed distance, v , above the sphere, with dynamic velocity h , set by force balance between viscous drag and buoyancy force proportional to the density difference between the mixed fluid, $p(t)$, and the ambient fluid profile $\bar{p}(X(t)2v)$.^{1.}

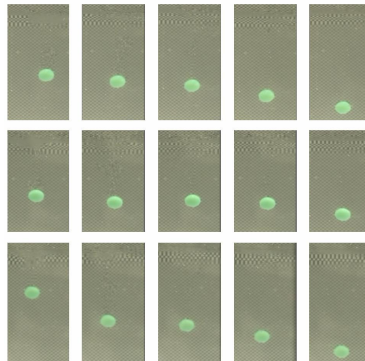
Diagram Detailing the Initial Conditions of Eqs. 1 and 2^{1.}

For each bead of specific density and volume, there may exist a "critical region" of bottom fluid densities in which the transverse motion exhibits extreme behavior as a function of layer thickness. Specifically, this is a window of bottom fluid densities in which over a series of at least 15 trials, there was a period of no transverse motion, followed by a period of transverse motion, followed by another period of no transverse motion. Since all factors affecting the bead motion are kept as constant as possible, the explanation of the varying behaviors could be explained by the elongation of the stratification layer thickness. As the layer thickness, defined as the distance between 10% and 90% of the total density shift between the top and bottom fluids, increased with time, data was taken for a range of different stratifications in each tank. This increase was due to both diffusion and the disruption caused by the bead and its retrieval device repeatedly passing through the transition layer. This data should help us determine the behavior of the ambient fluid profile term in the original coupled ODE's, $\bar{p}(X(t) - \omega)$.

The experimental set-up consists of a salt water bottom layer carefully controlled for density and temperature and a distilled water top layer. First the density and temperature of the bottom layer is fixed to approximately 1.03856 g/cc (for the 1.0396 bead), then the distilled water layer is carefully poured on top. The density of the distilled water is then confirmed to be .997 g/cc. The densities of both the top and bottom layers are established to an accuracy of .00003 g/cc using an Anton Paar Density Meter. The thickness of the density gradient between these two layers can then be measured using a conductivity probe to measure the distance from L10 to L90 (the points at which 10% and 90% of the density change has occurred respectively.) Once the tank is stratified, a bead is dropped from a submerged position into the tank. If the densities and layer thickness have been calibrated correctly the bead will slow down and then reverse direction entirely as it passes through the layer before slowly sinking to the bottom of the tank. Most experiments involve multiple drops which are video recorded and then reviewed on tape to determine whether or not the bead achieved transverse motion (defined as having negative velocity). Also, Data Tank bead tracking software is sometimes used to confirm the motion of the bead and to determine bead velocity throughout each trail.

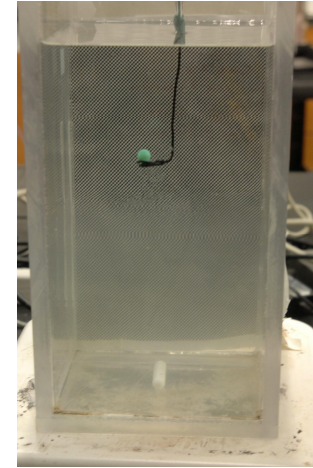


Sample Velocity vs. Depth Graph^{1.}



3 Part Photo Montage

- Row 1: Early stage drop, low layer thickness, no transverse motion.
- Row 2: Mid stage drop, medium layer thickness, transverse motion observed.
- Row 3: Later stage drop, large layer thickness, no transverse motion.

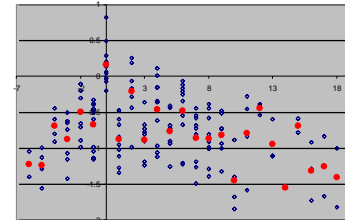


A Typical Tank



1.03936 Bead With Low Density Fluid Stem

It is theorized that the motion reversal is caused by the layer of lighter density fluid that surrounds the sphere as it passes through the transition layer. This entrained fluid remains associated with the sphere for several seconds after entering the higher density fluid. However, instead of diffusing, this fluid remains attached to the low density layer by a long stem of the same fluid reaching from the sphere to the transition layer. This stem can reach a length of greater than 12 cm, and always dissociates from the bead in a burst of released fluid likely caused by viscous drag.



Maximum Sphere Velocity vs. Drop Number

In this plot of Maximum Sphere Velocity vs. Drop Number, the Drop Number is directly related to the "thickness" of the transition layer. With increasing numbers of drops, the layer is affected by both diffusion and by the passage and retrieval of the sphere itself. Here, the drop data was taken for 15 tanks with bottom layer densities ranging from 1.03851-1.03856 g/cc, using a sphere with a density of 1.03639 g/cc. The data was shifted such that the zero point is located at the first bounce, indicated by a positive maximum velocity, to correct for the differences in initial layer thickness. The red spots indicate the average maximum velocity at each layer thickness.

Sphere Density (g/cc)	Sphere Radius (m)	Volume of Entrained Fluid (cm ³)	Ratio of Volumes of Entrained Fluid to Sphere
1.04011	0.002	0.00747	0.223
1.03936	0.005	0.08815	0.168
1.055	0.002	0.00817	0.224

In addition, it is interesting to note the relationship between the size and density of the sphere and the amount of entrained fluid associated with it. To determine this, video footage was analyzed of three beads immediately prior to a reversal of motion. The radius of the sphere was compared to that of the of the entrained fluid was measured and

References

- 1. N. Abaid et al, "An internal splash: Levitation of falling spheres in stratified fluids", Physics of Fluids, 5, 2 (2004).