Thin Film Evolution Undergraduate Research Computational Mathematics URCM

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Study of Thin Films

Thin films are:

* Thin material layer coating on a object
* Fluid layers thickness is much less than the lateral extent
* Microstructure (nanometer to micrometer in thickness)

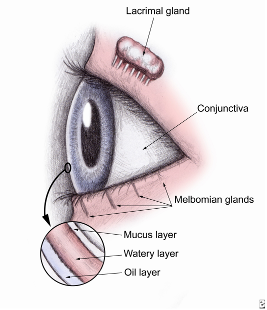
*** Goal : Study of Dry Eye Syndrome ***

- Porous layer
  Contact lens can be modeled as a porous layer (ref. Raad and Sabau)

Image courtesy of: http://www.emedicinehealth.com

Introduction to Tear Films

- Fluid Layers on the eye
  - Oil Layer
    60.3+/- 2.5 nm [1]
  - Watery Layer ('pre-lens' film)
    4 - 7μm [2]
  - Mucus Layer
    2 - 7μm [3]

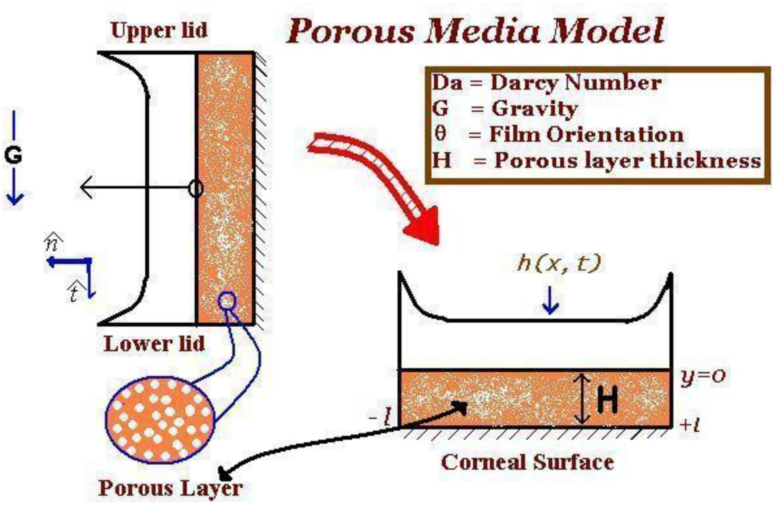


References:

1. Lynda Charters
2. D.Sullivan, D. Dartt, M. Meneray
3. Nichols, Chiappino, Dowson, M.D

Image courtesy of:
http://www.emedicinehealth.com

Eye Model Reconstruction



Introduction to Thin Films

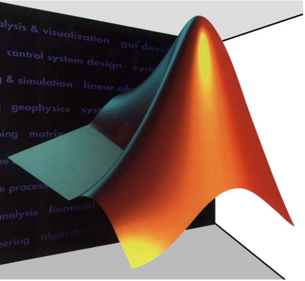
* Identify the behavior of thin film layer in time
* Factors: gravity, pressure of a porous layer, the presence of a porous layer underneath the thin liquid film and more...

NOTE:

We examine a modification of the Braun & Fitt (BF) Equation

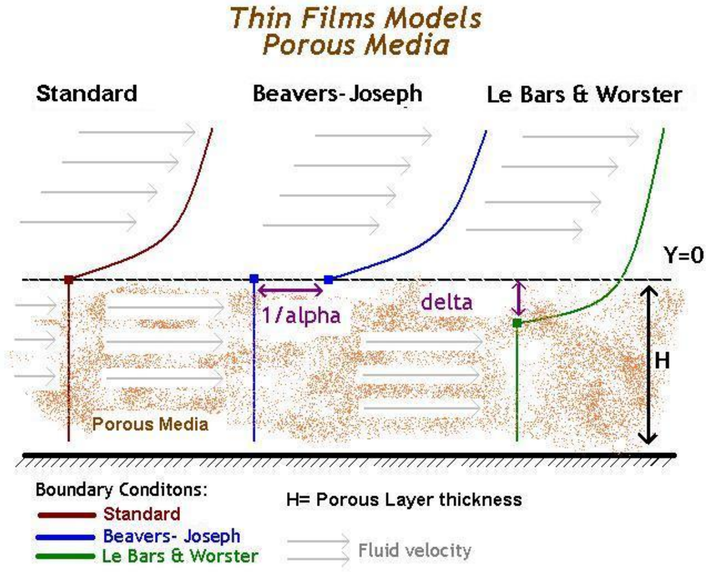
- Standard Case
- Beavers - Joseph Boundary Condition (BJ)
- Le Bars & Worster Boundary Condition (LW)

Numerical Method



- Finite difference method
  - Spatial Derivatives Representation
- Taylor series
  - One sided-derivative approximation
    - boundary condition
- Matlab Simulation
  - ODE solvers - ode23s, ode45
- FORTRAN Code
  - Faster computation

Porous Media Film



Thin Film Evolution Equations

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\left\{ f(h) * \left(\frac{1}{Ca}\frac{\partial^3 h}{\partial x^3} - Gy\cos\theta \frac{\partial h}{\partial x} + Gx\sin\theta \right) \right\}.$$

*** Note: Evaporation effect is not included ***

- Standard Case

$$f_{Standard} = \frac{h^3}{3} + Da(h+H)$$

- Beavers-Joseph

$$f_{BJ} = \frac{h^3}{3} + \frac{\sqrt{Da}}{\alpha}h^2 + Da(h+H)$$

- Le Bars & Worster

$$f_{LW} = \frac{(h+\delta)^3}{3} + Da(h+H)$$

Interests

Specific cases

1. All equations hold the result of BF equation
*Note: set Da$= 0$, and $1/\alpha$ or $\delta = 0$

$$f_{BF} = \frac{h^3}{3}$$

2. Darcy-Weisbach coefficient (Da) effects

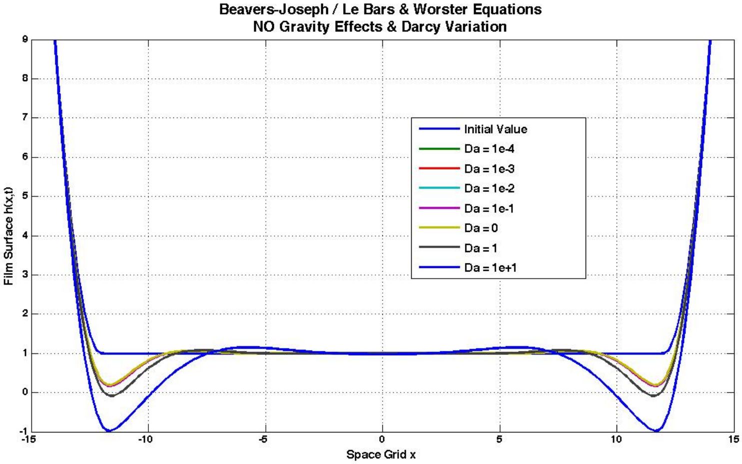
* Ratio of porous hole radius to film thickness

3. Slip of fluid velocity at the liquid/porous interface
*** α and δ

4. The film orientation
Ex: $\theta = \pi/2$ (up-right) or $\theta = 0$ (lay flat)

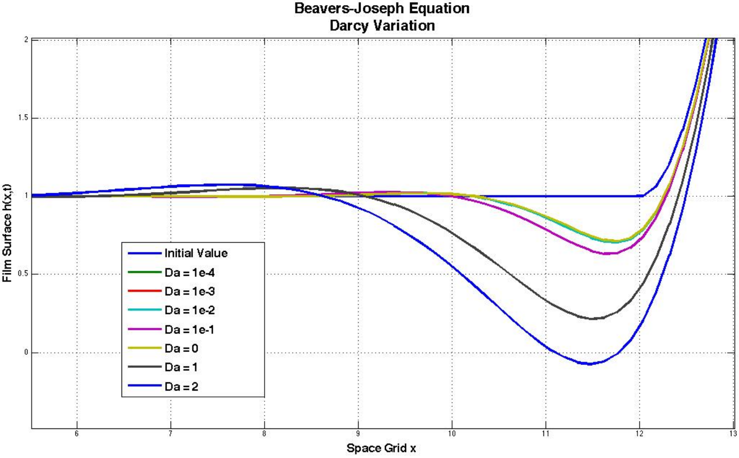
Main Result

- BF vs. BJ vs. LW Equations
  - $1/\alpha$ or $\delta = 0$, H$=$ 1, and Gx$=$ Gy$=$ 0



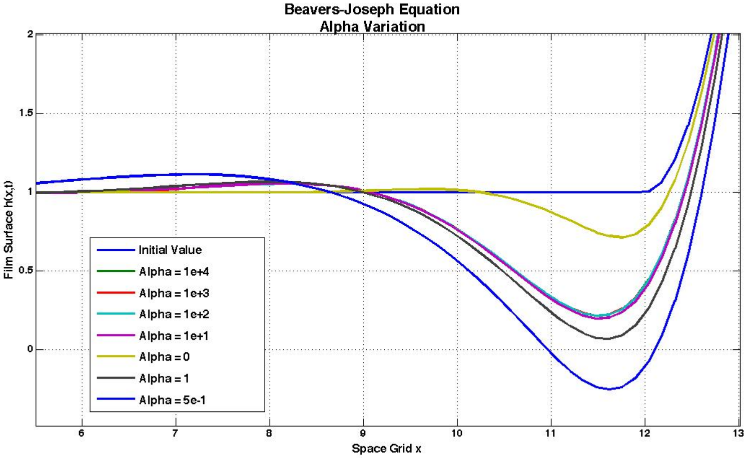
Main Result - con't

- Beavers-Joseph Boundary Condition
  - Variation of Da, $\theta = 0$, $1/\alpha = 0$, H$= 1$, and Gx$=$ Gy$= 0$



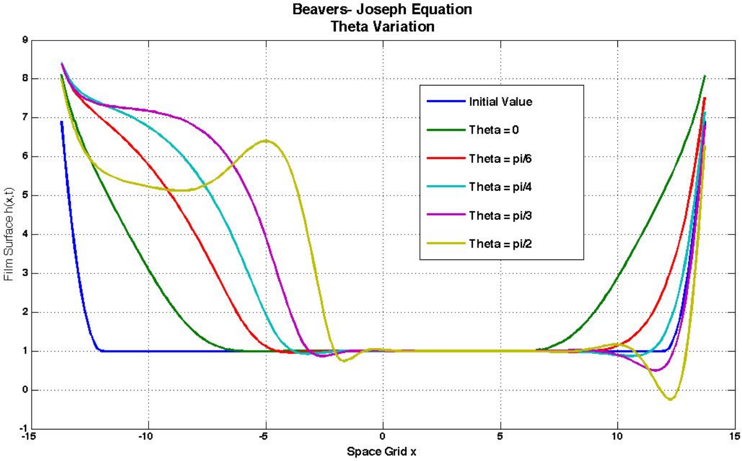
Main Result - con't

- Beavers-Joseph Boundary Condition
  - Variation $1/\alpha$, $\theta = 0$, Da$= 1$, H$= 1$, and Gx=Gy=0



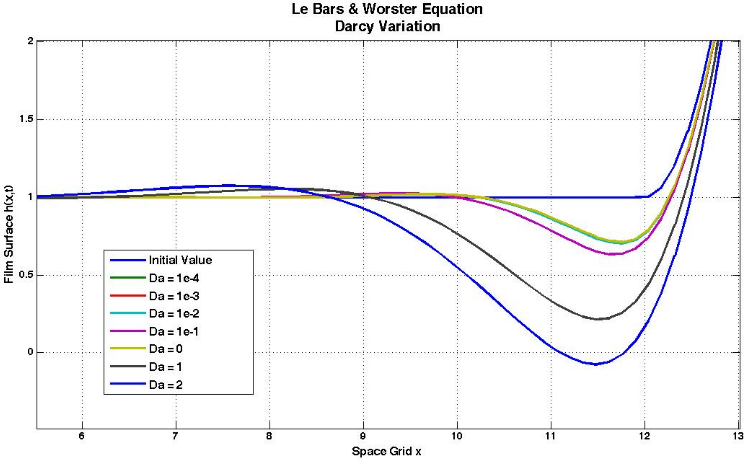
Main Result - con't

- Beavers-Joseph Boundary Condition
  - Variation θ, Da= 1, $1/\alpha$= 0, H= 1, and Gx= Gy= 1



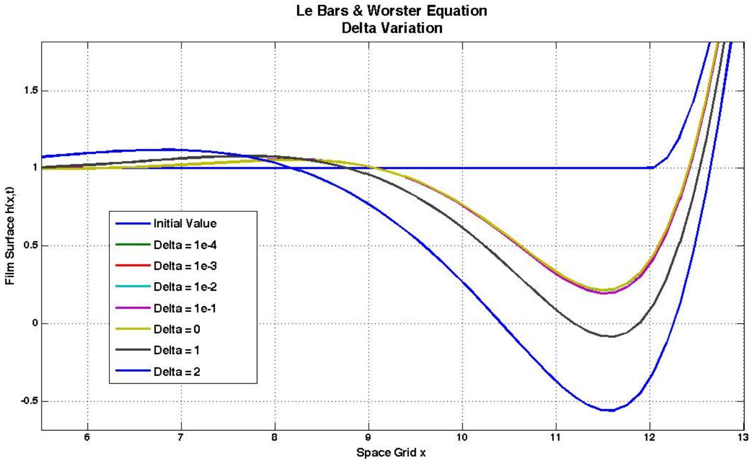
Main Result - con't

- Le Bars & Worster Boundary Condition
  - Variation of Da, $\theta = 0$, $\delta = 0$, H$= 1$, and Gx$=$ Gy$= 0$



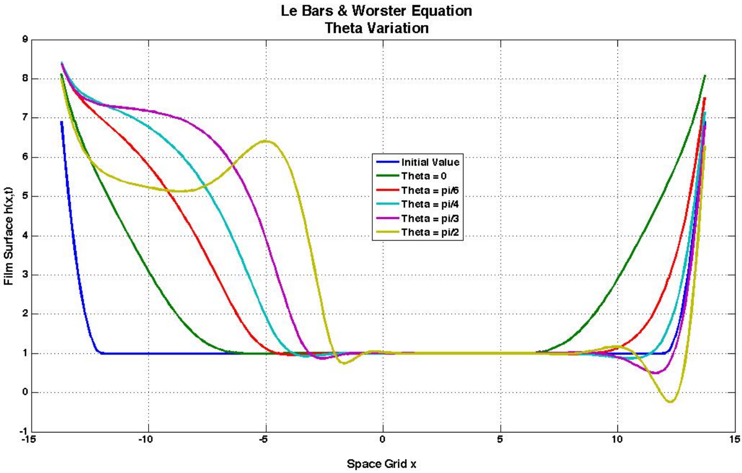
Main Result - con't

- Le Bars & Worster Boundary Condition
  - Variation δ, $\theta = 0$, Da$= 1$, H$= 1$, and Gx$=$ Gy$= 0$



Main Result - con't

- Le Bars & Worster Boundary Condition
  - Variation of θ, Da$=1$, $\delta=0$, H$=1$, and Gx$=$ Gy $=1$



Conclusion

* All equations, the models start to rupture as
  - Time increased
  - Darcy increased - Ratio of radius to film thickness
* Beavers-Joseph and Le Bars & Worster Equations:
  - Recovered Standard Case and Braun & Fitt models
  - α decreased (BJ) or δ increased (LW)
  - Angle (θ) and Gravity (Gx, Gy) effect the behavior of film motion

Summary

- Challenging problem
  - Duration of computation - speed
  - Data comparison
  - Translate to real life applications
- Solution
  - Convert code to FORTRAN
  - Numerical method- (Spectral Method)

Next steps:

- Evaporation effect
- Angle (θ) and Gravity (Gx, Gy) effect the behavior of film motion

Image courtesy of www.targetwoman.com

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Thank You

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