NC STATE UNIVERSITY

Climate Modeling Laboratory

Conservative Remapping with Adaptive Subgrid Approximations: Application to Conservative Semi-Lagrangian Transport

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INTRODUCTION: Conservative remapping is the process of transferring data from one mesh to another while conserving the global and local data integrals. The present study improves this process by providing a more accurate description of the sub-grid data distribution. Typically, the Piecewise Parabolic Method (PPM) with a pre-processing monotonic limiter is used to describe the sub-grid distribution. PPM is generally 4th-order accurate but degenerates at local extrema and steep jumps in the data. The Piecewise Hyperbolic Method (PHM) with a power limiter is generally less accurate than PPM but retains full accuracy at steep jumps.

OBJECTIVE: Develop an adaptive scheme which uses the PPM in the general case but uses the PHM for steep jumps and local extrema in an optimal formulation to give a robust accuracy improvement for smooth and non-smooth data while ensuring essential monotonicity, positive-definiteness, and conservation.

Cell-Integrated Semi-Lagrangian (CISL) Scheme

- Arrival cells bounded by thin black lines
- Original cell means (ρ_i) denoted by dotted black lines
- $x_{i\ 1/2}$ denotes a cell boundary

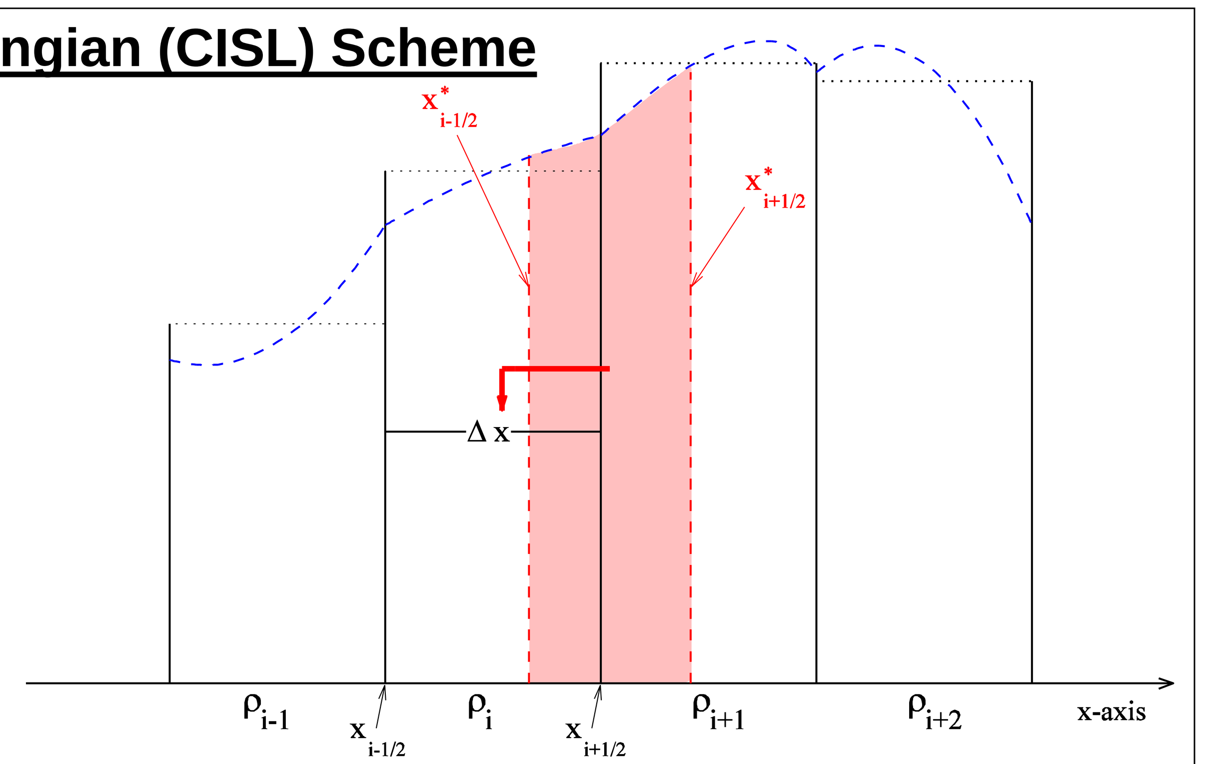
(1) Trace arrival cell boundaries upstream to find departure cell boundaries (red dashed lines)

(2) Fit 1-D approximations to arrival cells (blue dashed lines)

(3) Integrate over departure cell (light red shading)

(4) Replace arrival cell with integrated mass of departure cell

(5) Divide by arrival cell grid spacing for new value



Conservative Cascade Scheme (CCS)

- Arrival grid cells bounded by thin black lines
- Departure grid cells bounded by thin red lines

(1) Trace arrival cell boundaries upstream

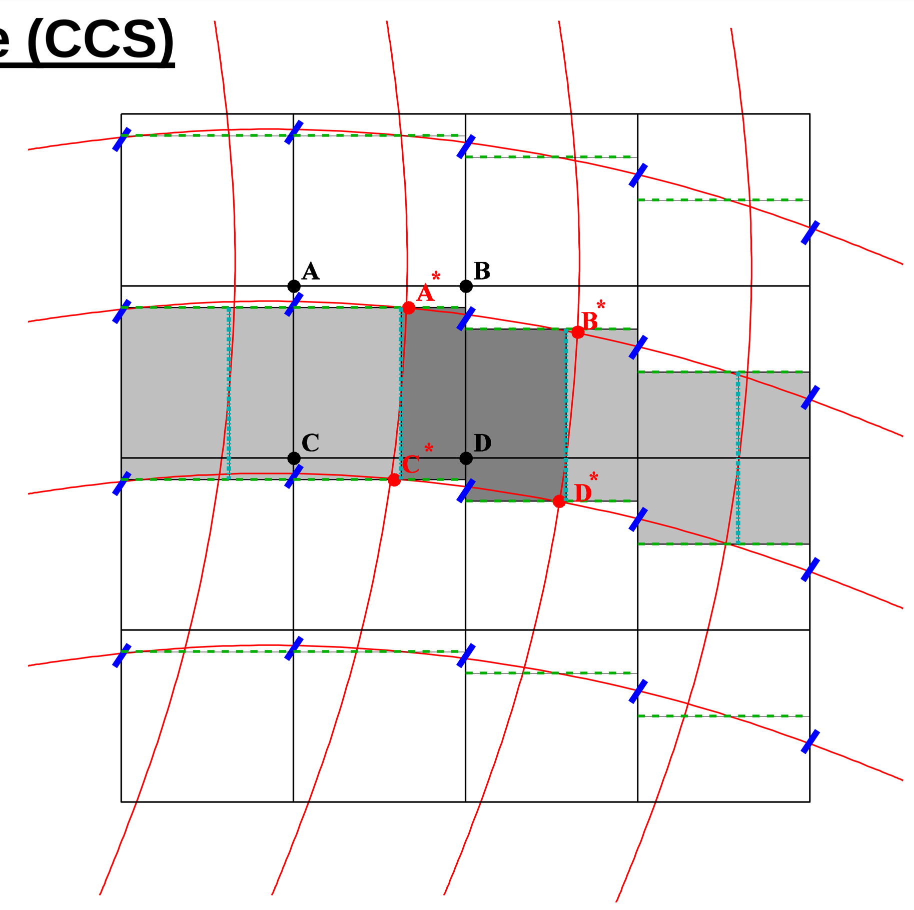
-Black grid to red grid; **ABCD** to **A*B*C*D***

(2) Use intersections between arrival x-lines and departure y-lines (dark blue slashes) to form intermediate grid

Only performed once for multiple transported scalars

(3) For each column, perform 1-D CISL parallel to arrival y-lines

(4) For each row, perform 1-D CISL parallel to departure x-lines



The Sub-grid Approximations

Piecewise Parabolic Method (PPM)

(1) Uses 4th-order approximated cell interface values

(2) Generally 4th-order accurate

(3) Monotonically limited by altering interface values

(4) 1st-order accurate at local extrema

(5) Arbitrary accuracy loss at steep jumps

Piecewise Hyperbolic Method (PHM)

(1) Uses 2nd-order approximated cell interface derivatives

(2) Generally 3rd-order accurate

(3) Monotonically limited via power limiter

(4) 1.5-order accurate at local extrema

(5) Remains 3rd-order accurate at steep jumps

Piecewise Parabolic Method - Hybrid (PPM-H)

PPM used for smooth data

PHM adaptively used for steep jumps

No consistent improvement for extrema

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1-D Constant Advection of a Sine Wave

Simulation Details

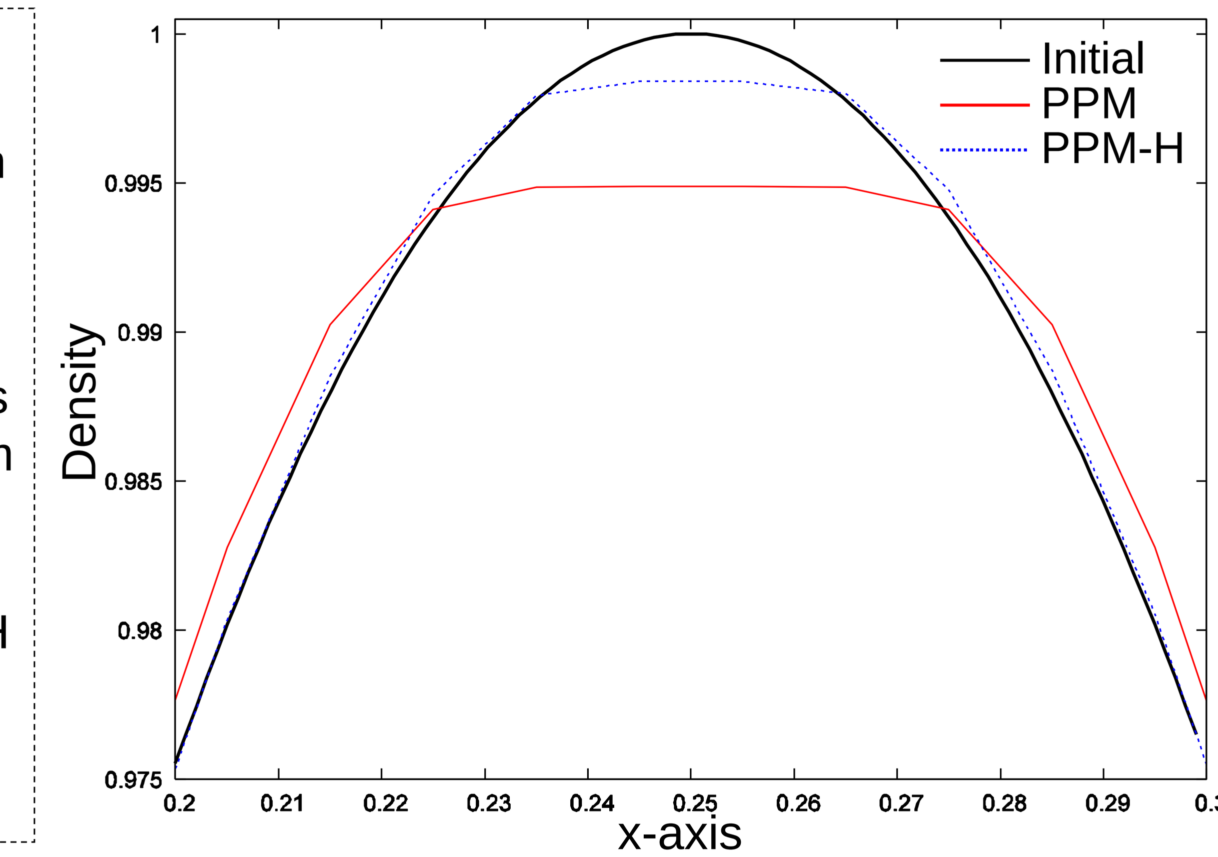
- Cyclic advection of sine wave
- Uniform fluid flow at 1 m s^{-1}
- 100-cell regular mesh
- 10 revolutions
- Time step: 0.5 s
- Courant Number: 0.5

Error Comparison

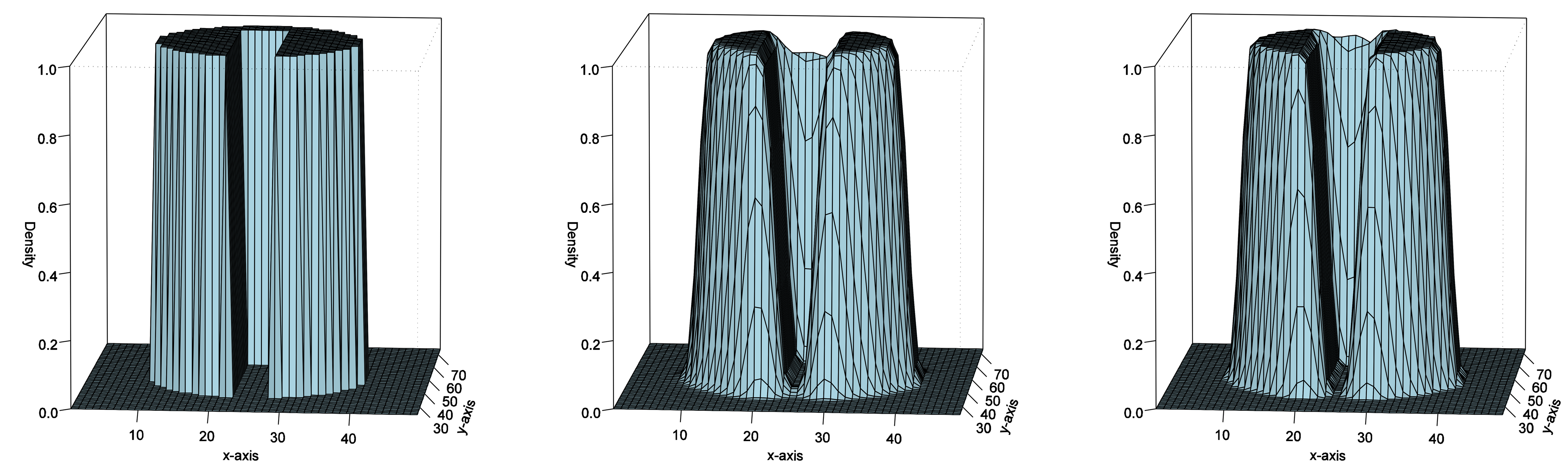
- PPM L_1 Error: 1.46 x 10^{-3}
- PPM-H L_1 Error: 4.93 x 10^{-4}

Remarks

- PPM has a well-known problem of clipping smooth extrema.
- PPM-H resolves extrema more accurately and keeps the slope surrounding them steeper.
- For smooth flows, PPM-H has been found to be as much as 5 times more accurate than PPM.



Cartesian Solid Body Rotation of a Slotted Cylinder



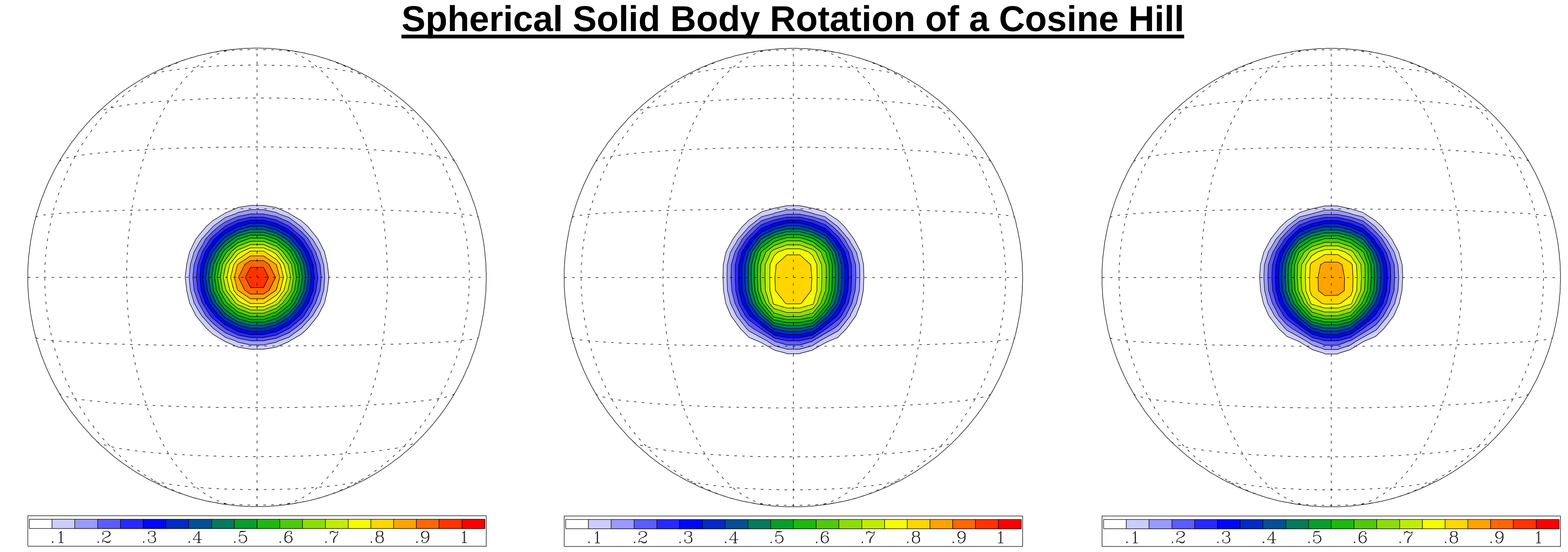
(a) Initial Conditions

(b) PPM after 1 revolution L_1 Error: 0.187

(c) PPM-H after 1 revolution L_1 Error: 0.170

Solid body rotation in Cartesian geometry consists of rotating the entire domain a constant angular velocity about its center point. In this case, the domain is rotated for one revolution over a 101 x 101 cell mesh with 1 m grid spacing for 96 time steps at an angular velocity of 3.64 x 10^{-5} s^{-1} and a time step of 1800 s. All figures are displayed on the same truncated domain for enhanced detail.

Spherical Solid Body Rotation of a Cosine Hill



(a) Initial Conditions

(b) PPM after 1 revolution L_1 Error: 8.08 x 10^{-2}

(c) PPM-H after 1 revolution L_1 Error: 5.34 x 10^{-2}

In this case, the domain is rotated poleward about an equatorial axis for one revolution over a 128 x 65 cell mesh with 2.8 grid spacing for 256 time steps (12 days) at an angular velocity of 6.06 x 10^{-6} s^{-1} and a time step of 4050 s.

Conclusion

The adaptive method, PPM-H, robustly improves upon the accuracy PPM for conservative semi-Lagrangian transport both smooth and non-smooth data in 1-D, 2-D Cartesian, and 2-D spherical contexts These improvements may apply to other remapping procedures such as conservative interpolation and remapping of Lagrangian coordinates to a reference grid.