

Model of Air-Driven Mucus Clearance Mechanisms in Lungs

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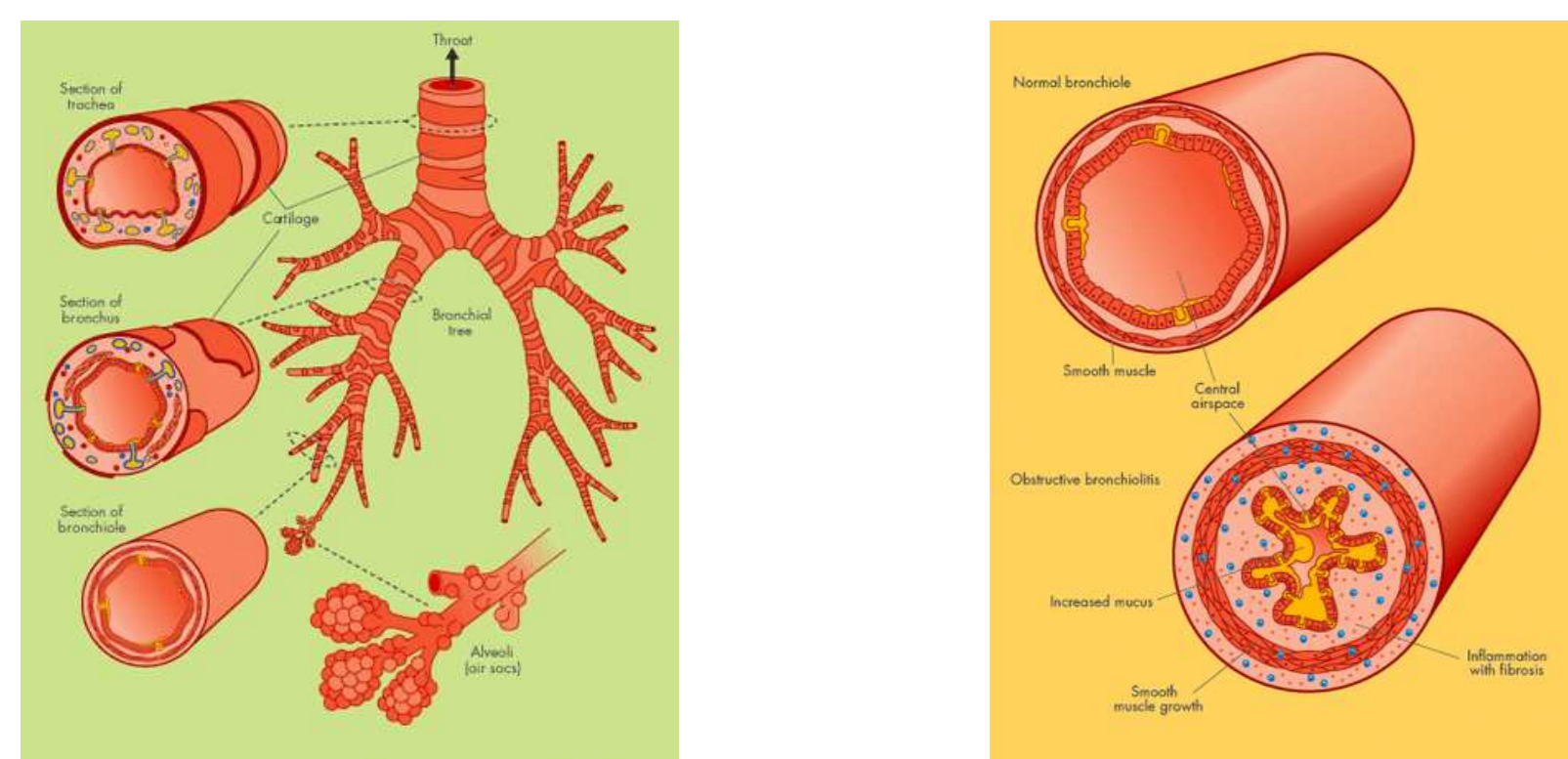


Abstract

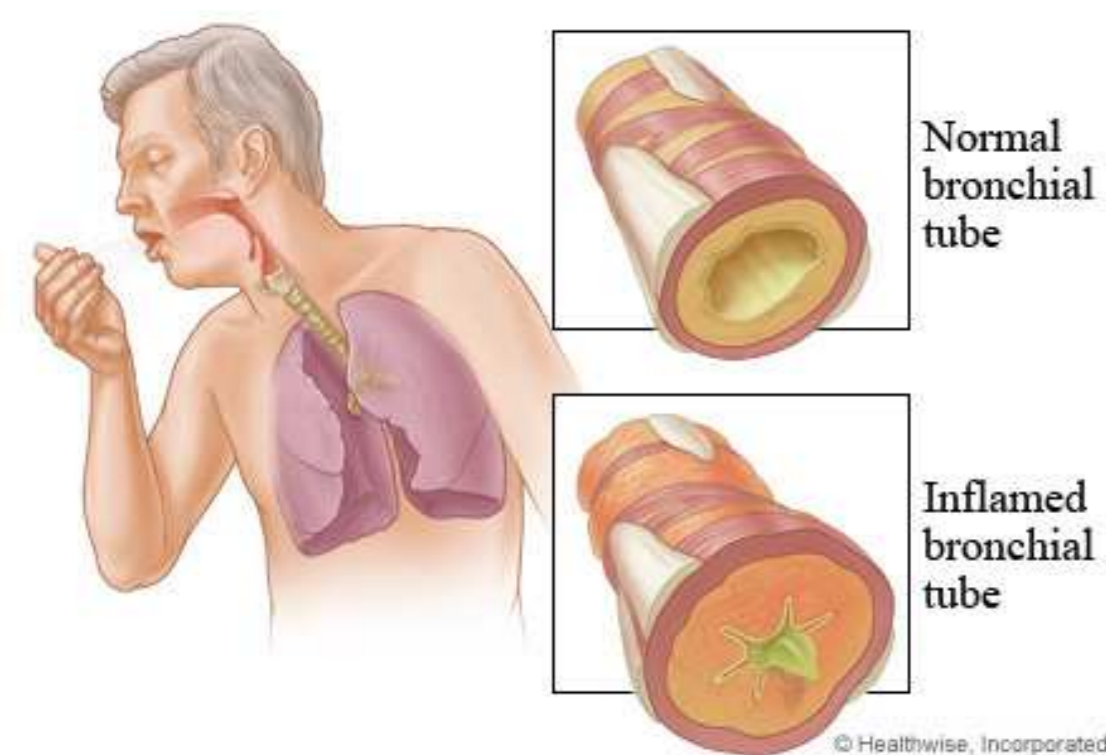
Air flow through lung airways interacts with their mucus coating layer, which has much higher viscosity than air, providing an important mechanism for mucus motility and clearance of pathogens from the lung. To understand this process, we have developed a model for examining the stability of two superimposed fluids with very different viscosity in the two layers. This instability can be quite different from that occurring with a single fluid, and its study much more challenging. To shed some light on it, we have adopted a perturbation approach. Our focus is on the limit of large viscosity contrast, which has not been fully analyzed or understood yet. Here the asymptotic properties of stability will be investigated. In this parameter regime, we expect our results to have a direct application in understanding air-driven mucus clearance mechanisms and the mucus transport phenomena in human lungs.

1 Motivation and introduction

The lungs and the respiratory system behave as the interface between air and human body: these inhale air and then exchange oxygen. Mucus clearance defends the lungs against inhaled bacteria. Pathogens are usually trapped by lung mucus and cleared by mucus transport. Since it is critical to keep the lung system functional and healthy, hydrodynamic transport is very important for the mucus clearance.



Note: The above pictures are found at <http://www.abpi.org.uk/>



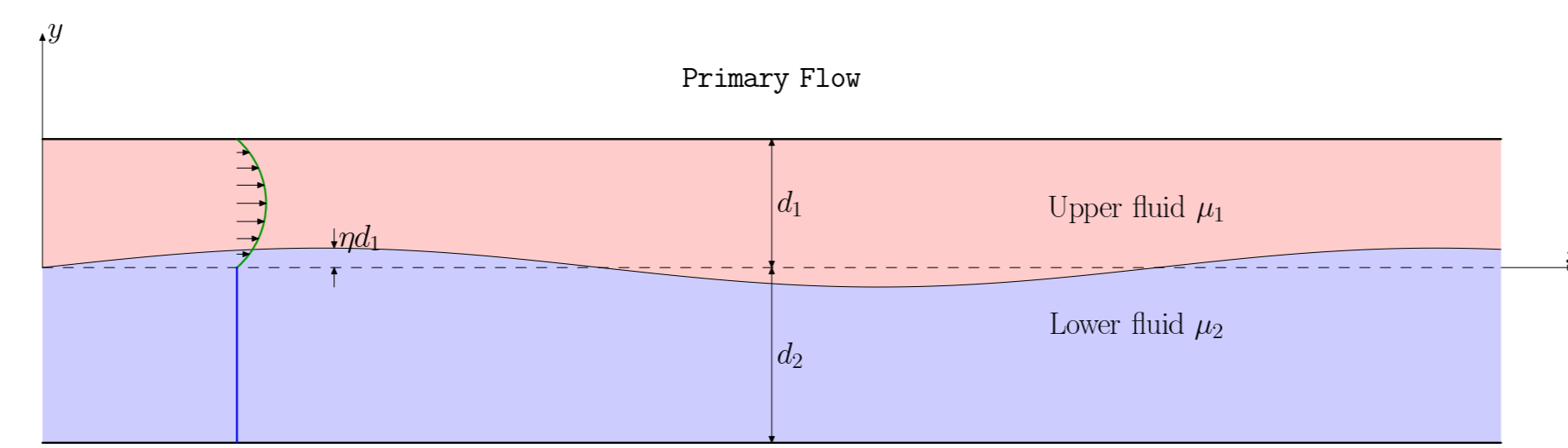
Basic 2-dimensional wave interaction mechanism between the laminar air-flow and mucus layer in a rigid channel is investigated. The purpose is to explore the stability of two layers of immiscible and incompressible viscosity stratified flows in the limit of large viscosity contrast, since the viscosity contrast of the mucus and the air is about 10^4 . The results will have a direct application to air-driven mucus clearance mechanisms in lungs.

The two-dimensional motion of the viscous fluid in each layer is governed by the Navier-Stokes equation:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho_j} \nabla p + \frac{\mu_j}{\rho_j} \Delta \mathbf{u} \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0$$

supplemented by boundary conditions and continuity conditions at the interface. The subscript $j = 1, 2$ refer to the upper and lower flow. The following figure is the configuration of the problem and the primary flow.



The primary flow can be determined under the assumption that it has only horizontal velocity component u , which is independent of t and x . Furthermore, the incompressible viscous fluids are under a constant pressure difference, i.e. $dp/dx = -K = \text{constant}$.

Let

$$m = \frac{\mu_2}{\mu_1}, \quad h = \frac{d_2}{d_1}, \quad r = \frac{\rho_2}{\rho_1},$$

η = the deviation of interface from its mean position.

The velocity distributions for the primary flow are

$$u_1 = -\frac{K}{2\mu_1} y^2 + a_1 y + b,$$

$$u_2 = -\frac{K}{2\mu_2} y^2 + a_2 y + b,$$

where

$$b = \frac{d_1^2 h(1+h)K}{2(h+m)\mu_1}, \quad a_1 = \frac{d_1 K(m-h^2)}{2(h+m)\mu_1}, \quad a_2 = \frac{d_1 K(m-h^2)}{2m(h+m)\mu_1}.$$

2 Formulation of the problem

The motion is resolved into the primary motion and the perturbation motion. $\hat{u} = U_j + u', \hat{v} = v', \hat{p} = P + p'$, in which U_j is the dimensionless primary velocity and P is the dimensionless pressure for the primary flow. We use the stream function ψ , in terms of which

$$u' = \psi_y, \quad v' = -\psi_x.$$

Assume an exponential time factor for all perturbation quantities, and write

$$(\psi, p') = \{\phi(y), f(y)\} \exp\{i\alpha(x - c\tau)\},$$

in which $c = c_r + ic_i$.

The stability or instability is then decided by the sign of c_i .

- If $c_i < 0$, then the solution is a stable linear eigenmode.
- If $c_i > 0$, then the solution is an unstable linear eigenmode, which means that the amplitude of the disturbance grows exponentially with time.
- If $c_i = 0$, then it is a neutrally stable solution.

Choose d_1 as the characteristic length, the mean velocity of the upper flow V as the characteristic velocity and the Reynolds number $R = \frac{\rho_1 V d_1}{\mu_1}$, the dimensionless linearized system is: For the upper fluid

$$\phi^{iv} - 2\alpha^2 \phi'' + \alpha^4 \phi = i\alpha R \left\{ (U_1 - c) (\phi'' - \alpha^2 \phi) - U_1' \phi \right\} \quad (2)$$

For the lower fluid, we write χ instead of ϕ

$$\chi^{iv} - 2\alpha^2 \chi'' + \alpha^4 \chi = i\alpha R m^{-1} r \left\{ (U_2 - c) (\chi'' - \alpha^2 \chi) - U_2' \chi \right\} \quad (3)$$

The imposed boundary and interfacial conditions are :

- The no-slip and no-penetration boundary conditions on the upper and lower boundaries
 - The continuity of the velocity at the interface
 - The continuity of the stress at the interface
- Totally, there are eight coupled boundary and interfacial conditions.

Instability of Plane Poiseuille Flow of one Viscous Fluid

Orr-Sommerfeld equation is the dimensionless equation governing stability of one viscous fluid

$$\phi^{iv} - 2\alpha^2 \phi'' + \alpha^4 \phi = i\alpha R \left\{ (U - c) (\phi'' - \alpha^2 \phi) - U'' \phi \right\} \quad (4)$$

$$\phi(\pm 1) = 0, \quad \phi'(\pm 1) = 0$$

where $U = 1 - y^2$ is the primary flow.

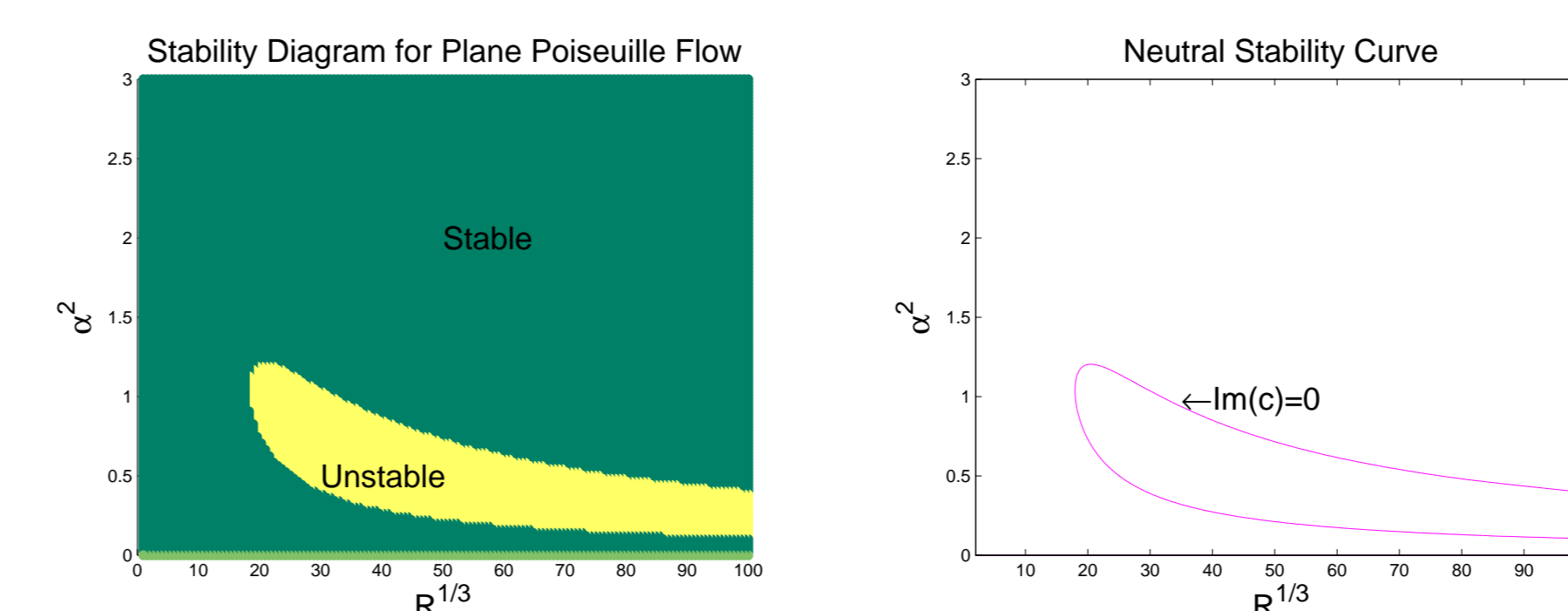
Using the Chebyshev method[2], we seek an approximate numerical solution of (4) of the form

$$\phi(y) = \sum_{n=0}^N a_n T_n(y)$$

where $T_n(\cos \theta) = \cos(n\theta)$ is the n th-degree Chebyshev polynomial of the first kind.

Equations for the expansion coefficients a_n are deduced by substituting the above formula into (4) and we get the linear matrix system for the eigenvalue problem.

We reproduce the ‘‘thumb curve’’ for Orr-Sommerfeld equation as the figures below.



3 Asymptotic expansion

Suppose the solutions of the stability governing equations (2) and (3) have regular asymptotic expansions, then

$$\phi(y) = \phi_0(y) + \varepsilon^\gamma \phi_1(y) + \dots$$

and

$$\chi(y) = \chi_0(y) + \varepsilon^\delta \chi_1(y) + \dots,$$

where the small number $\varepsilon = \frac{1}{m}$ and $\gamma, \delta > 0$.

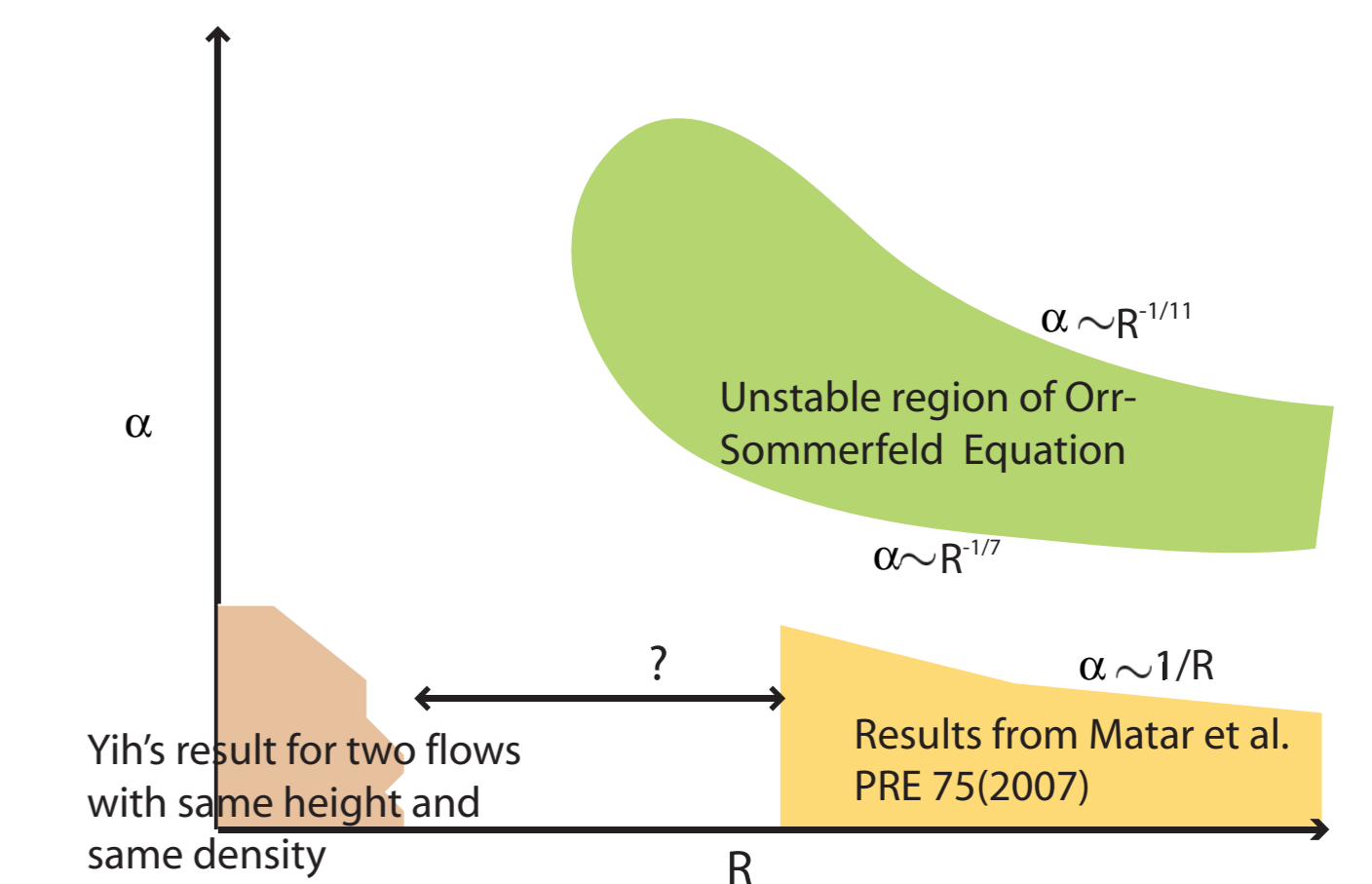
As an extension of Yih's results, we consider that the wave number $\alpha \sim \varepsilon^\beta$ ($0 < \beta < 1$), which implies that the wavenumber is small but relatively large compared to ε in the limit of high viscosity contrast $m \rightarrow \infty$.

When the viscosity contrast $m \rightarrow \infty$, the interesting cases we have investigated are

		Reynolds number R	
		$O(1)$	∞
$O(\varepsilon)$	Trivial solution		Leading order: Orr-Sommerfeld like equation Higher order approximations depend on the order of αR
$O(1)$	Leading order: Orr-Sommerfeld equation Higher order approximations: Orr-Sommerfeld like equation		Rayleigh like equations show up

- If $\alpha R \sim O(\varepsilon^\kappa)$ ($\kappa > 0$), the regular perturbation gives the trivial solution.
- For the case that $\alpha R \sim O(1)$ or $\sim O(\varepsilon^{-\kappa})$ ($0 < \kappa < 1$), the leading order is Orr-Sommerfeld equation of which we only know the asymptotic solution with high Reynolds number.

The following sketch shows the known results related to this problem. The colored regions mean the unstable region.



- From C. S. Yih's long-wave asymptotic results, we know that the stratified plane Poiseuille flow is unstable when $h = 1, r = 1$ and $m \sim O(1)$. When $m \rightarrow \infty$, his analysis breaks down.
- In Matar et al.'s paper, they derive an equation for the interface to get the stability results of the stratified flow with high viscosity contrast.
- Our work is to understand this problem with mild Reynolds number but high viscosity contrast.

4 Future work of 3D core-annular Poiseuille flow

Finally, we want to implement our stability result to the mucus clearance mechanism. The airflow passes through the lung airways coated by mucus fluid layers, which is stability problem of 3D core-annular Poiseuille flow with a large viscosity ratio. The coupled airflow-mucus system in the bronchial tube is different from the lubricated pipeline, in which the core is occupied by fluid with higher dynamical viscosity and which has been studied previously.

5 Acknowledgment

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References

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