

### Abstract

Curve matching has a variety of applications in computer image processing and image recognition. Two curves are equivalent if they lie in the same orbit. Two curves in  $\mathbb{R}^2$  are equivalent under the action of the Euclidean group if one curve can be mapped to the other by a combination of rotations, reflections, and translations. Differential invariants for the Euclidean group are well known and can be used to solve the curve matching problem. The use of differential invariants is problematic though because derivatives are sensitive to noise. We instead use integral invariants, which are much less sensitive to noise.

### References

- [1] Helmut Alt, Christian Knauer, and Carola Wenk. Comparison of distance measures for planar curves. *Algorithmica*, 38(1):45–58, 2004.
- [2] Michael Artin. *Algebra*. Prentice Hall Inc., Englewood Cliffs, NJ, 1991.
- [3] Irina Berchenko and Peter J. Olver. Symmetries of polynomials. *J. Symbolic Comput.*, 29(4-5):485–514, 2000. Symbolic computation in algebra, analysis, and geometry (Berkeley, CA, 1998).
- [4] Eugenio Calabi, Peter J. Olver, Chehrzad Shakiban, Allen Tannenbaum, and Steven Haker. Differential and numerically invariant signature curves applied to object recognition. *Int. J. Comput. Vision*, 26(2):107–135, 1998.
- [5] Eugenio Calabi, Peter J. Olver, and Allen Tannenbaum. Affine geometry, curve flows, and invariant numerical approximations. *Adv. Math.*, 124(1):154–196, 1996.
- [6] Mark Fels and Peter J. Olver. Moving coframes. I. A practical algorithm. *Acta Appl. Math.*, 51(2):161–213, 1998.
- [7] Mark Fels and Peter J. Olver. Moving coframes. II. Regularization and theoretical foundations. *Acta Appl. Math.*, 55(2):127–208, 1999.
- [8] Mark Fels and Peter J. Olver. Moving frames and coframes. In *Algebraic methods in physics (Montréal, QC, 1997)*, CRM Ser. Math. Phys., pages 47–64. Springer, New York, 2001.
- [9] S. Feng, I. Kogan, and H. Krim. Integral invariants for 3D curves: an inductive construction. In *Proceedings of IS&T/SPIE Joint Symposium*, page 11p, San Jose, CA, 2007.
- [10] S. Feng, I. A. Kogan, and H. Krim. Classification of curves in 2D and 3D via affine integral signatures. *Submitted*, 2008. Preprint is available at <http://arxiv.org/abs/0806.1984v1>.

- [11] Mark L. Green. The moving frame, differential invariants and rigidity theorems for curves in homogeneous spaces. *Duke Math. J.*, 45(4):735–779, 1978.
- [12] P. Griffiths. On Cartan’s method of Lie groups and moving frames as applied to uniqueness and existence questions in differential geometry. *Duke Math. J.*, 41:775–814, 1974.
- [13] C. E. Hamm and M. S. Hickman. Projective curvature and integral invariants. *Acta Appl. Math.*, 74(2):177–193, 2002.
- [14] Irina A. Kogan. Two algorithms for a moving frame construction. *Canad. J. Math.*, 55(2):266–291, 2003.
- [15] Irina A. Kogan and Peter J. Olver. The invariant variational bicomplex. In *The geometrical study of differential equations (Washington, DC, 2000)*, volume 285 of *Contemp. Math.*, pages 131–144. Amer. Math. Soc., Providence, RI, 2001.
- [16] S. Manay, D. Cremers, Byung-Woo Hong, A.J. Yezzi, and S. Soatto. Integral invariants for shape matching. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 28(10):1602–1618, Oct. 2006.
- [17] Peter J. Olver. *Applications of Lie groups to differential equations*, volume 107 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1993.
- [18] Peter J. Olver. *Equivalence, invariants, and symmetry*. Cambridge University Press, Cambridge, 1995.
- [19] Peter J. Olver. *Classical invariant theory*, volume 44 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 1999.
- [20] Jun Sato and Roberto Cipolla. Affine integral invariants for extracting symmetry axes. 15(8):627–635, 1997.