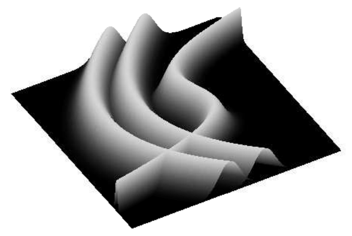
The dynamics of immersed boundaries in viscoelastic fluids

J.C. Chrispell, R. Cortez, D.B. Khismatullin, L.J. Fauci
Center for Computational Science, Tulane University, New Orleans, LA 70118



ABSTRACT: Many biological fluids are viscoelastic and require a nonlinear constitutive equation to describe the evolution of the extra-stress tensor. We use an immersed boundary framework to model processes that involve the movement of immersed elastic boundaries interacting with a surrounding viscoelastic fluid. We present recent results on applications including dynamics of a closed membrane moving under surface tension, and phase-locking of swimming sheets.

Viscoelastic Fluid Model

The time dependent Giesekus model for viscoelastic fluid flow:

$$\boldsymbol{\sigma} + \lambda\left(\frac{\partial \boldsymbol{\sigma}}{\partial t} + \mathbf{u}\cdot\nabla\boldsymbol{\sigma} - (\nabla\mathbf{u})\boldsymbol{\sigma} - \boldsymbol{\sigma}(\nabla\mathbf{u})^T + \varepsilon\boldsymbol{\sigma}^2\right) - 2\alpha\mathbf{d}(\mathbf{u}) = \mathbf{0} \quad \text{in } \Omega \quad (1)$$

$$Re\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) + \nabla p - (1-\alpha)\Delta\mathbf{u} - \nabla\cdot\boldsymbol{\sigma} = \mathbf{f} \quad \text{in } \Omega \quad (2)$$

$$\nabla\cdot\mathbf{u} = 0 \quad \text{in } \Omega \quad (3)$$

$$\frac{\partial \mathbf{X}(\xi,t)}{\partial t} = \int_\Omega \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{X}(\xi,t))\,d\mathbf{x}, \quad (4)$$

where

$$\mathbf{f}(\mathbf{x},t) = \int_\Gamma \mathbf{F}(\mathbf{X},t)\delta(\mathbf{x}-\mathbf{X}(\xi,t))\,d\xi \quad \text{and} \quad \mathbf{d}(\mathbf{u}) = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right).$$

- The fluids velocity, pressure and extra stress are denoted by $\mathbf{u}, p$, and $\boldsymbol{\sigma}$ respectivly. The fluids Weissenberg and Reynolds number are written as λ and Re, $\alpha \in (0,1)$ denotes the fraction of the total viscosity that is viscoelastic, and ε is the Giesekus nonlinear parameter.
- A force density $\mathbf{F}$ describes the forces on a boundary.
- The boundary position is described in a Lagrangian frame by $\mathbf{X}(\xi,t)$.
- Interpolation and spreading operations are used to couple the Lagrangian boundary with the Eulerian description of the fluid flow.
- A second order finite difference projection method is used to solve the fluid equations.

Closed Membrane Moving Under Surface Tension

An immersed membrane initially expressed as

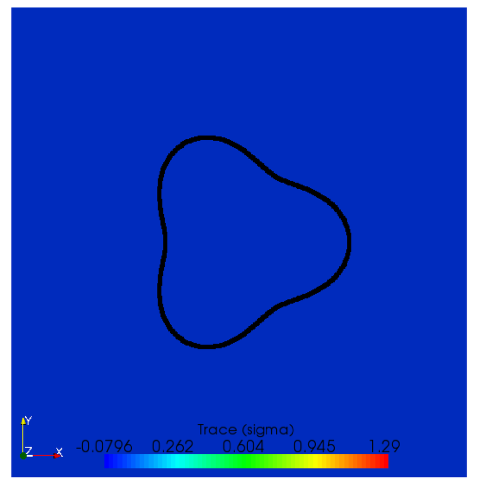
$$r(\theta) = 1 + A\cos(n\theta) \quad (5)$$

is moved by forces set based on the curvature

$$\kappa(\theta) = \frac{(r^2 + 2(r')^2 - rr'')}{((r)^2 + (r')^2)^{\frac{3}{2}}}$$

such that

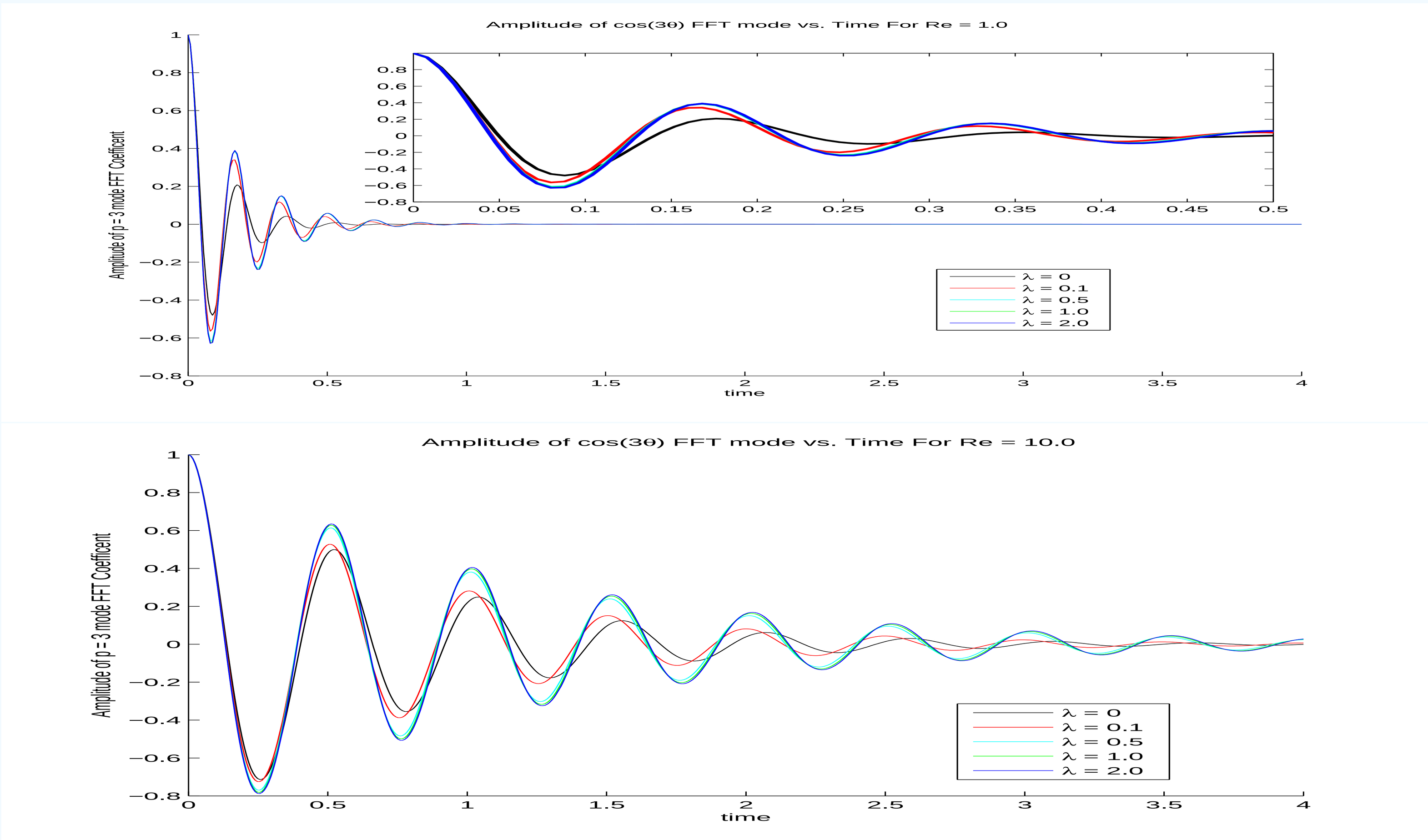
$$\mathbf{F}(\theta) = \sigma\kappa(\theta)\mathbf{n}.$$



- Here $\mathbf{n}$ is the outward normal and σ is a material parameter.
- In all the computations shown $\sigma = 25, A = 0.2$, and $n = 3$.
- The plot at the left shows the initial position of a membrane.
- Further work on closed membranes moving under surface tension may be found in [1,2].

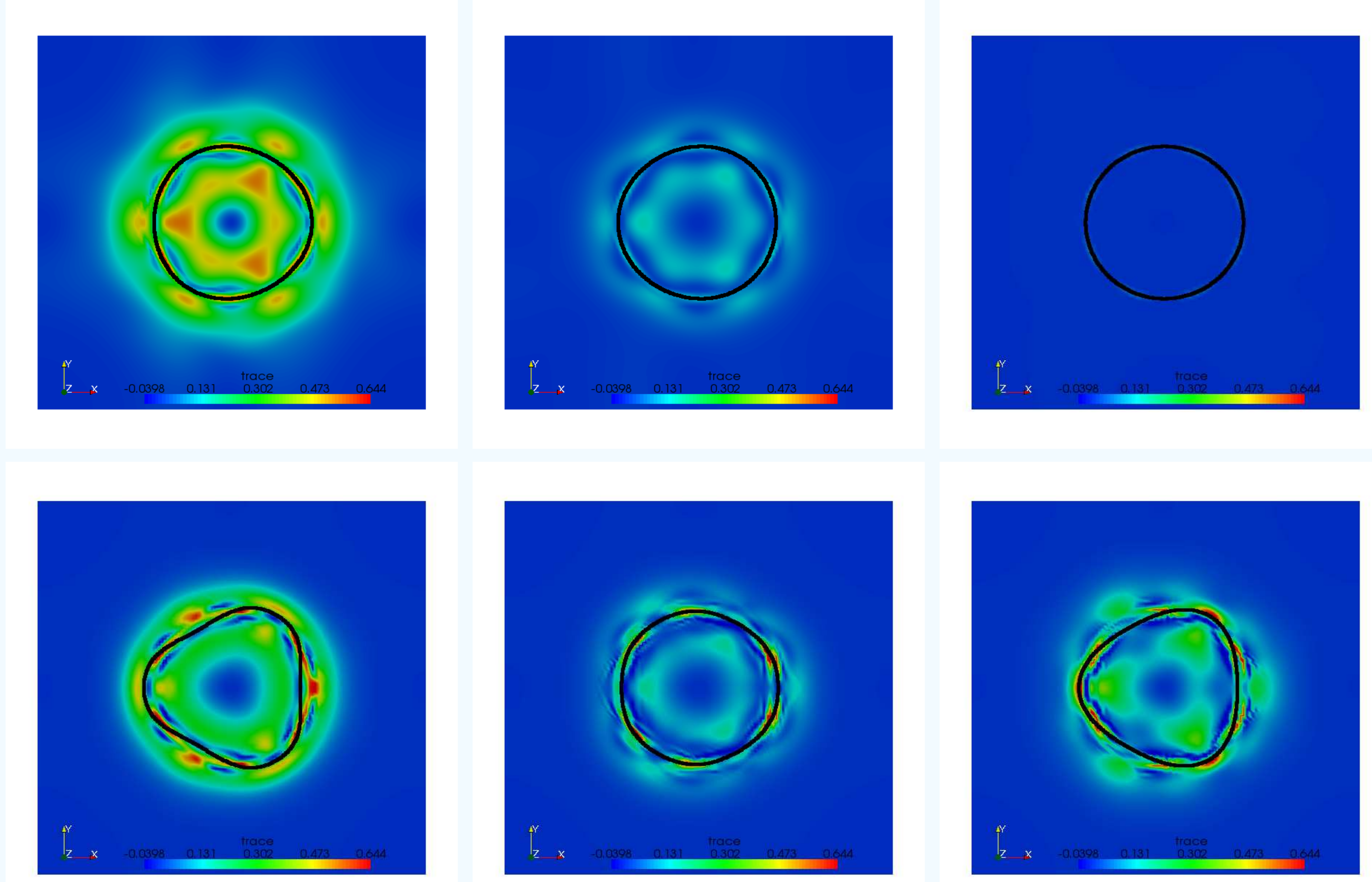
Viscoelasticity Increases Membrane Oscillations

The amplitude of the $\cos(3\theta)$ FFT mode is tracked for equally spaced points on cubic splines of the discretized membrane. The plots below show how a fluids Reynolds and Weissenberg number effect the membrane's damping rate and oscillation frequency.



Evolution of the Extra stress

The following table of figures show the $tr(\boldsymbol{\sigma})$, representing the mean-squared distension of the polymer coils in the viscoelastic fluid for different values of Re, λ, and t.

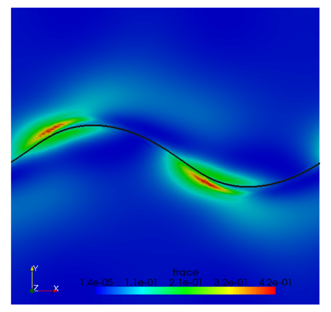


Swimming Sheets

We investigate the swimming velocity of material points on a sinusoidal sheet propagating a traveling wave of the form:

$$y = b\sin(\kappa x + \omega t + \phi)$$

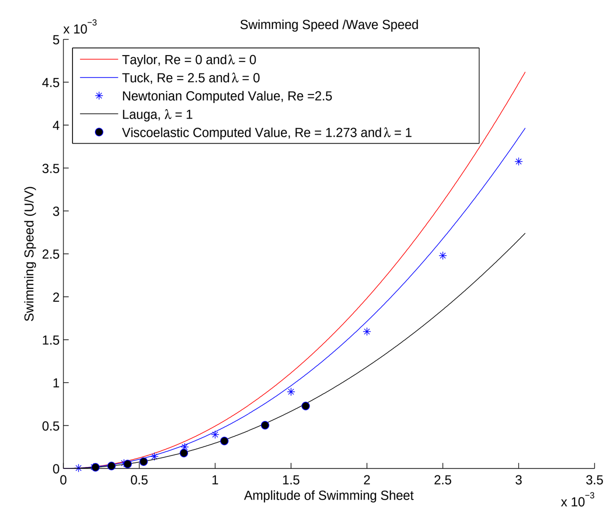
in a periodic domain. The plot at the right shows the $tr(\boldsymbol{\sigma})$ for a sheet in a fluid with $Re = 2$ and $\lambda = 0.25$.



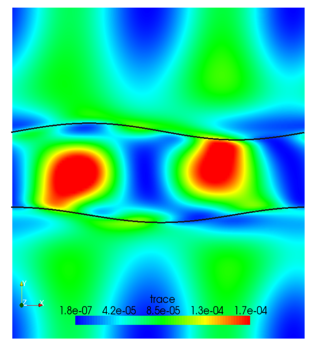
Swimming Speeds

G.I. Taylor [4], E.O. Tuck [6], and E. Lauga [3] showed that the number of wavelengths travelled per period, $\frac{U}{V}$, for an infinite swimming sheet in different fluids is given by the expressions below. The Figure at the right shows our numerical experiments compared with the asymptotic formulas.

- Stokes (Taylor, 1951):
$$\frac{U}{V} = \frac{1}{2}b^2\kappa^2\left(1 + \frac{19}{16}b^2\kappa^2\right) + O(b^6)$$
- Navier-Stokes (Tuck, 1968):
$$\frac{U}{V} = \frac{1}{4}b^2\kappa^2\left(1 + \frac{1}{F(Re)}\right) + O(b^4\kappa^4)$$
- Viscoelastic (Lauga, 2007):
$$\frac{U}{V} = \left(\frac{1+\lambda^2(1-\alpha)}{1+\lambda^2}\right)\left(\frac{1}{2}\omega b^2\kappa\right) + O(\omega b^4\kappa^3)$$



Phase Locking of Swimming Sheets



The plot at the left shows the $tr(\boldsymbol{\sigma})$ for two out of phase swimming sheets almost immediately after the simulation was started in a fluid with $Re = 0.32$ and $\lambda = 1$. Our goal in future work is to simulate full phase locking of swimming sheets in viscoelastic fluids. This work is an extension of the work done on swimming sheets in [5].

[1] R. Cortez and D. A. Varela. The dynamics of an elastic membrane using the impulse method. *J. Comput. Phys.*, 138(1):224–247, 1997.
[2] D. Khismatullin and A. Nadim. Shape oscillations of a viscoelastic drop. *Physical Review E*, 63, 2001.
[3] E. Lauga. Propulsion in a viscoelastic fluid. *Physics of Fluids*, 19, 2007.
[4] G.I. Taylor. *Proc. R. Soc. Ser*, A 209:447, 1951.
[5] J. Teran, L. Fauci, and M. Shelley. Viscoelastic fluid response can increase the speed and efficiency of a free swimmer. *To Appear: Physics Review Letters*, 2009.
[6] E.O. Tuck. A note on a swimming problem. *Journal of Fluid Mechanics*, 31:305–308, 1968.

Research partially supported by the NSF under grant no. DMS-0652795.

email: jchrispe@tulane.edu