NEW MINIMAL REPRESENTATION OF SELF PROPELLED SWIMMERS IN STOKES FLOW USING REGULARIZED FUNDAMENTAL SOLUTIONS

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ABSTRACT

•We develop a new representation of self propelled swimmers in low Reynolds number viscous incompressible flow that efficiently and effectively captures collective dynamics in free space and in the presence of a wall/boundary.

•The representation developed uses regularized fundamental solutions of the Stokes equation and is derived so as to retain the fluid flow features produced by an organism but using only one or two singularity elements. We call these "minimal swimmer" representations.

•It is an extension of previous work to introduce a new way of breaking down the structure of an organism when studying suspensions, by accurately depicting the flow pattern observed in their vicinity for both, representative pushers (e.g. E.Coli) and pullers (e.g. Chlamydomonas).

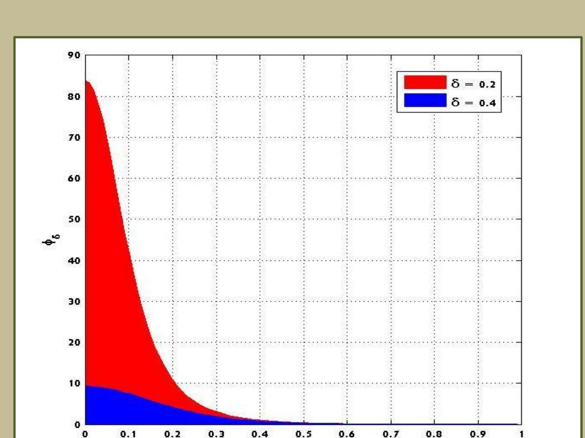
•The motility of suspensions of organisms and their interactions give rise to recurring regions of re-circulation and whirls. These regions exist even when the inertial forces are negligible and play a key role in transport of nutrients or other solutes suspended in the system. This study has applications in Biofilms and transport in micro-fluid devices and taxis problems.

Non -dimensional Model Equations:

$$-\nabla P + \Delta\vec{u} + f\varphi_\delta(\vec{x}-\vec{x}_0) = 0$$

$$\nabla\cdot\vec{u} = 0$$

Blob function:



$$\varphi_\delta = \frac{15\,\delta^4}{8\,\pi(r^2+\delta^2)^{\frac{7}{2}}}$$

$$r = |\vec{x}-\vec{x}_0|$$

Regularized Stokeslet in free space :

$$\vec{u}(\vec{x}) = \vec{f}\,H_1(r) + \vec{f}\cdot(\vec{x}-\vec{x_0})(\vec{x}-\vec{x}_0)H_2(r)$$

$$H_1(r) = \frac{B_\delta{}'(r)}{r} - G_\delta(r)\,,\; H_2(r) = \frac{r\,B''_\delta(r) - B'_\delta(r)}{r^3}$$

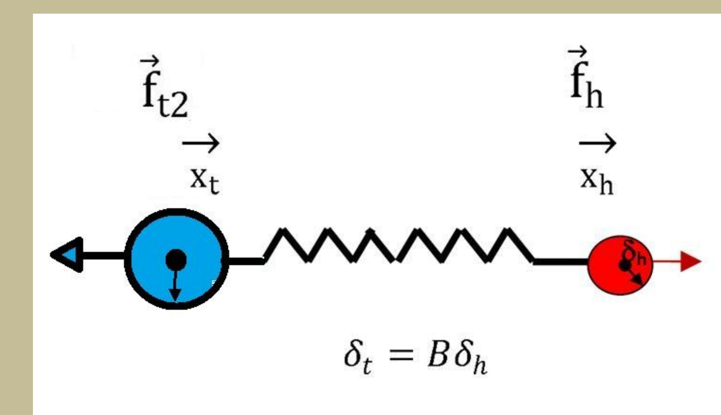
$$\Delta G_\delta(r) = -\varphi_\delta(r), \Delta B_\delta(r) = -G_\delta(r)$$

Regularized Potential Dipole in free space :

$$\mathbf{U}_{pd}(\mathbf{x}) = PD_\delta[\mathbf{q}] = \mathbf{q}D_1(r) + (\mathbf{q}\cdot\mathbf{x})\mathbf{x}D_2(r)$$

$$D_1(r) = \frac{G'_\delta}{r} - \psi_\delta, \qquad D_2(r) = \frac{rG''_\delta - G'_\delta}{r^3}$$

Minimal Representation derivation



$$\begin{aligned}\vec{f}(l)S(\vec{x}-\vec{x}_h) - \vec{f}(l)S(\vec{x}-\vec{x}_t) &= -\vec{f}(l)[S(\vec{x}+\frac{l}{2}\hat{\beta}-\vec{x}_0,\delta_t) - S(\vec{x}-\frac{l}{2}\hat{\beta}-\vec{x}_0,\delta_h)] \\ &= -l\vec{f}(l)\frac{[S(\vec{x}+\frac{l}{2}\hat{\beta}-\vec{x}_0,\delta_t) - S(\vec{x}-\frac{l}{2}\hat{\beta}-\vec{x}_0,\delta_h)]}{l}\end{aligned}$$

In the limit as l approaches zero we have the One Point representation as below

$$\vec{q}[\frac{C}{2}\frac{\partial}{\partial\delta}S(\vec{x}-\vec{x}_0,\delta) + \frac{C}{2}\frac{\partial}{\partial\delta}S(\vec{x}-\vec{x}_0,\delta)] - \vec{q}(\hat{\beta}\cdot\nabla)S(\vec{x}-\vec{x}_0,\delta)$$

$$\delta_h = \delta - \frac{Cl}{2} \text{ and } \delta_t = \delta + \frac{Cl}{2}$$

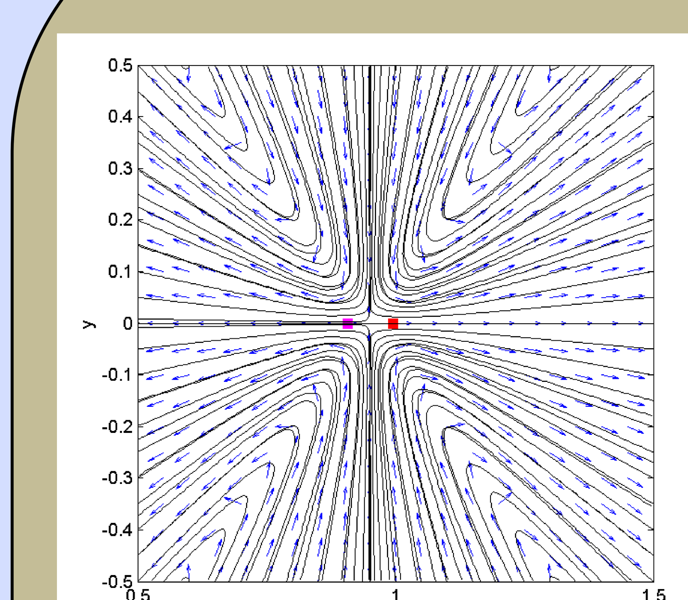
The asymmetric term comes from a Potential Dipole derived from a blob that is related to the Stokeslet blob by the following relationship

where

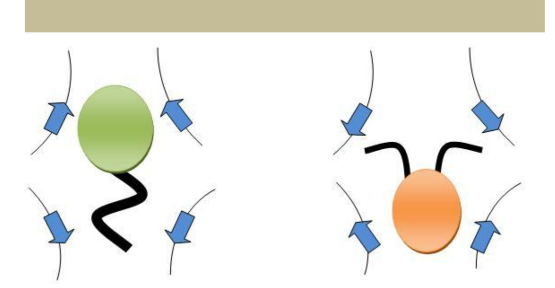
$$K\Phi'_d = -\frac{r}{\delta}[G''_s + \frac{2}{r}G'_s] = -\frac{r\Psi_s}{\delta}$$

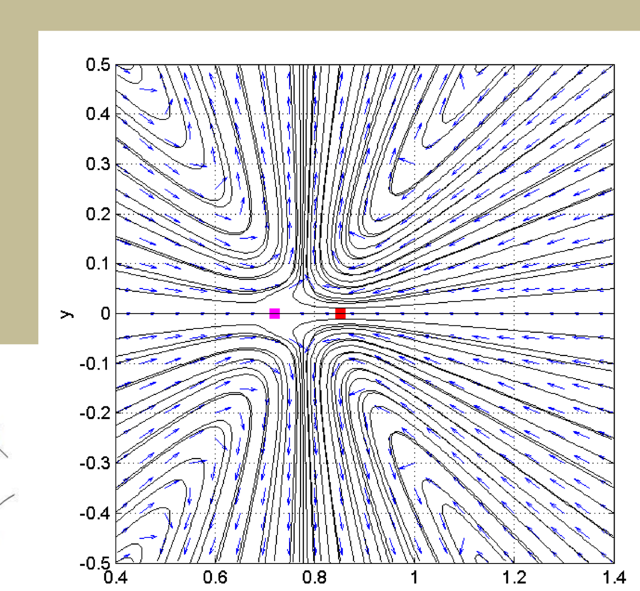
$$\vec{U}(\vec{x}) = -\vec{q}C\frac{\partial S}{\partial\delta} - \vec{q}(\hat{\beta}\cdot\nabla)S$$

Fluid flow using the One-point Formulation

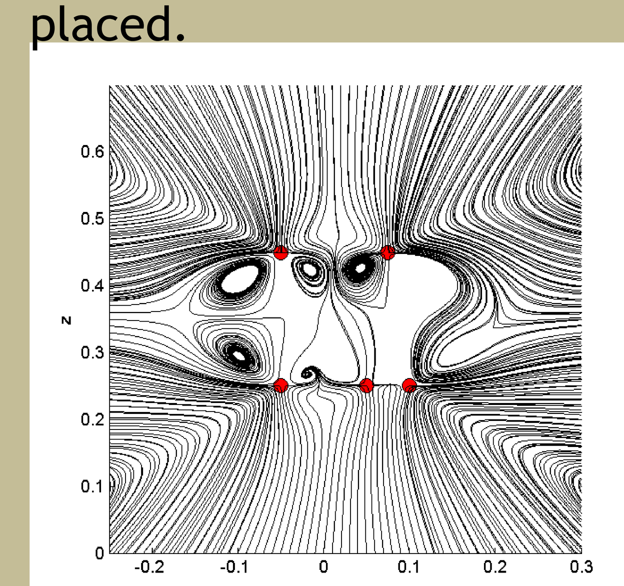


Capture flow due to Pushers and Pullers by a variation in the parameter C

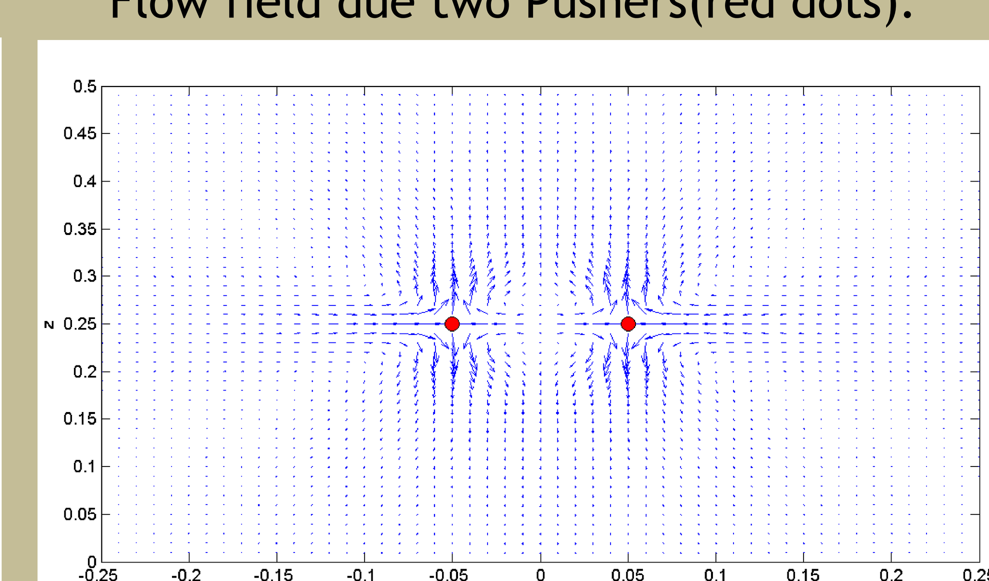




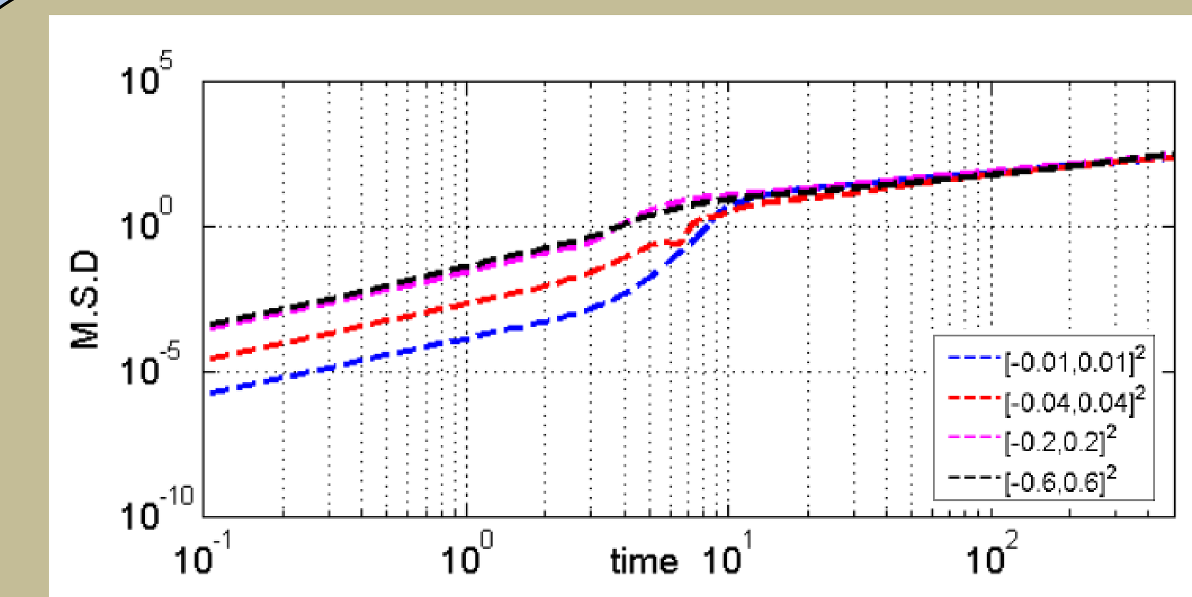
Streamlines in the flow due to five pushers(red dots) randomly placed.



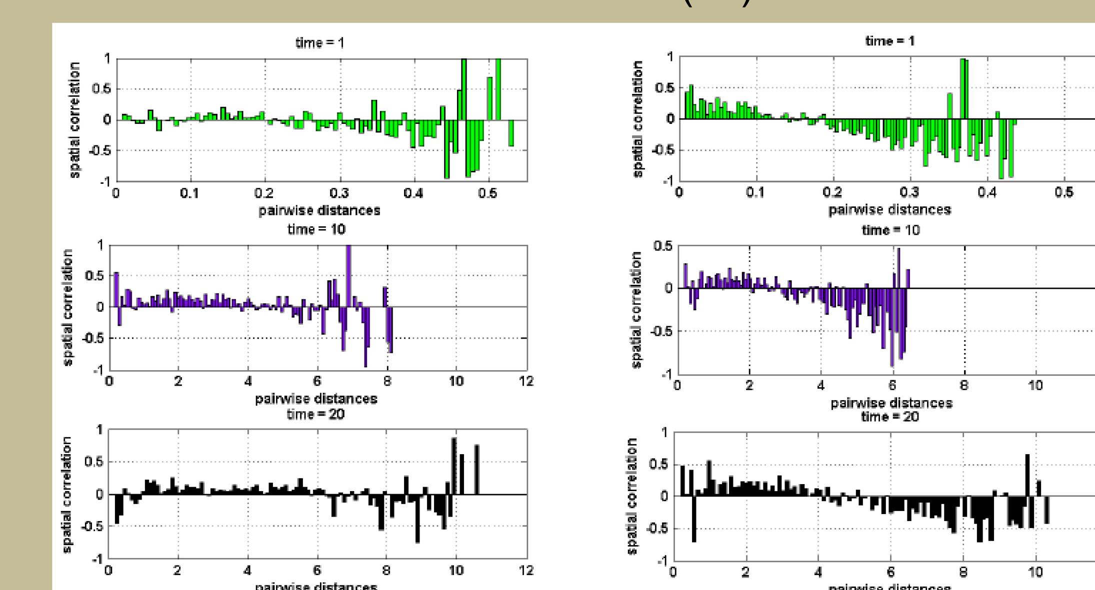
Flow field due two Pushers(red dots).



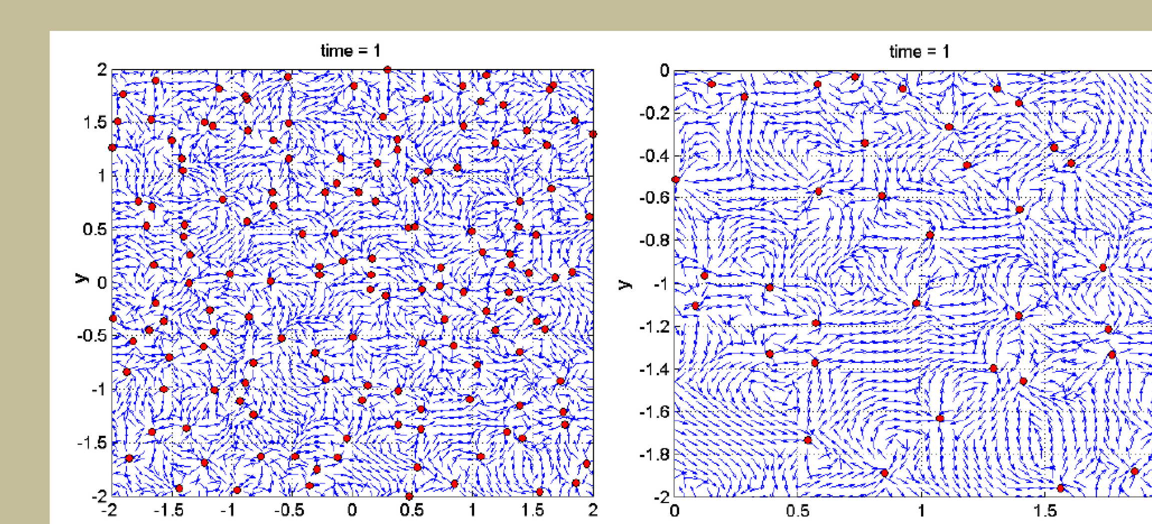
Some Collective Flow Results



Mean Squared Displacement vs. time, shows a ≈ t^2 behavior for short time and normal diffusion for (≈ t) later time.



Spatial Velocity Correlation for Pushers(R), is large positive for smaller pair wise distances than Pullers (L).



Collective Flow (Pushers) due to 400 organisms in fluid C = 0.001, L = 0.1, δ = 0.025.

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