THE NUMERICAL ASPECTS OF PARAMETER ESTIMATES FOR NONLINEAR VISCOELASTIC MODELS

J. MCGINNIS, B. DANDURAND, I. VIKTOROVA
DEPARTMENT OF MATHEMATICAL SCIENCES
CLEMSON UNIVERSITY

Abstract. The Volterra theory of heredity finds its applications in various branches of mathematical physics. The presented approach is based on the nonlinear hereditary type relationship between stresses, strains and time in visco-elastic solids – materials with memory. It can be modeled by the second type of Volterra's equation.

$$\varphi(\varepsilon) = \sigma + \int_0^t K(t-\tau)\sigma(\tau)d\tau. \tag{1}$$

It has been shown that such an equation can describe rather successfully the wide range of materials tested including polymers, composites, and metals. The higher the rate of loading, the closer the stress-strain diagram $\sigma \sim \varepsilon$ to the model $\varphi(\varepsilon)$. This model is an upper bound for the whole region of possible deformation of the material under the consideration.

The choice of kernel for the integral operator in equation (1) is the subject of several objective considerations. Physical and mathematical adequacy are the dominant ones. The exponential of arbitrary order function

$$\ni_\alpha(\beta, t) = \sum_{n=0}^{\infty} \frac{(\beta)^n t^{n(1-\alpha)}}{\Gamma[(1-\alpha)(n+1)]}$$

suggested for the kernel $K(t-\tau)$ presents the most general type to satisfy the above listed constraining considerations.

This research emphasizes the numerical aspects to the optimal procedure for parameter estimates for equation (1) with kernel

$$K(t) = \lambda \sum_{n=0}^{\infty} \frac{(\beta)^n t^{n(1-\alpha)}}{\Gamma[(1-\alpha)(n+1)]}.$$

The parameter α is known as a singularity parameter and can be determined from the isochronic diagrams from the creep testing. The other two parameters β and λ (and a third mechanical parameter ε_0) were estimated using the numerical procedure based on the minimization of the functional

$$F(p(\Delta s)) = \sum_{i=1}^{n} \left[\frac{\varepsilon[t_i, p(\Delta s)] - \varepsilon[t_i, p^*(\Delta s)^*]}{\varepsilon[t_i, p^*(\Delta s)^*]} \right]^2 \tag{2}$$

where $p(\Delta s)$ is the set of desired parameters and Δs is a choice of interval in the Laplace domain.

We will also present experimental data from creep and quasi-static loading tests.