Diffusion-Induced Bias In Near-Wall Velocimetry

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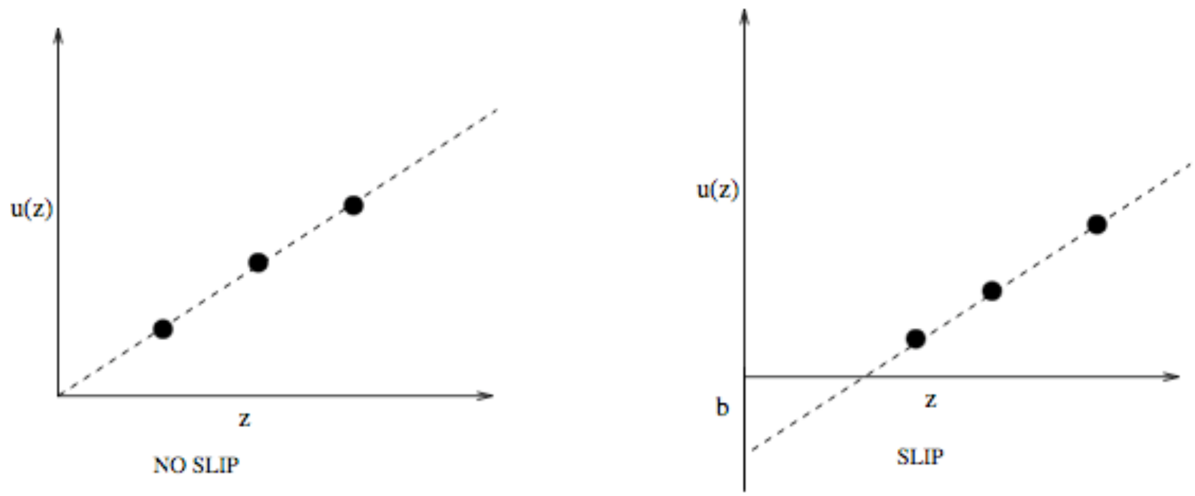
Introduction

- Work done with R. Sadr, H. Li and M. Yoda at Georgia Tech and P.J. Mucha.
- Experimental and numerical studies have shown that the no-slip boundary condition does not hold for 1-D flow past a solid stationary surface at micro- and nano-scales.
- Experimental difficulties in the near-wall region when using particle tracers.
- Possible explanation: electrostatic repulsion, but not convincing enough?
- We consider the effect of Brownian fluctuations influenced by a single wall on near-wall velocimetry using colloidal tracers.

Determination of slip length



- Basic idea: The measured velocity U_M is a good approximation of the sampled velocity $\langle u(z) \rangle$.
- Shear flow: $u(z)=G(z+b)$ with b slip length $\Rightarrow \langle u(z) \rangle$ depends on $\langle z \rangle$.
- Usually $\langle z \rangle = z_C$ the centre of the window.

Fokker Planck

- Let $D_\infty = kT/(6\pi\mu a)$ be the Stokes-Einstein diffusion coefficient. The wall hindered the diffusion as (Bevan-Prieve):

$$D_\perp(z) = D_\infty \frac{6z^2 - 10az + 4a^2}{6z^2 - 3az - a^2}$$

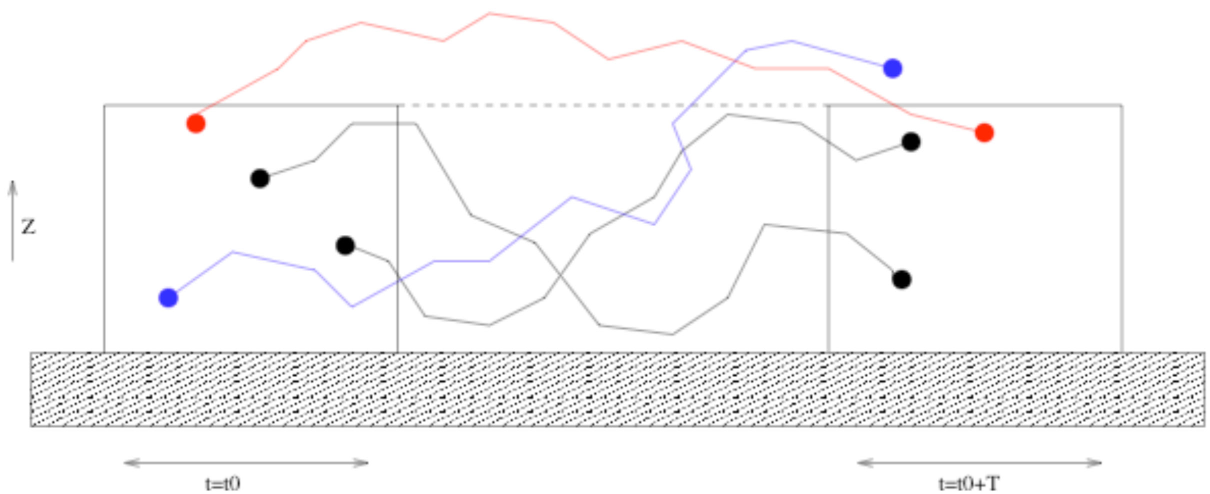
- Let *f(z,t)* be the average number density of particles at *z* at time *t* starting from an initially-uniform distribution in [*a*,*Z*].
- FP for the motion normal to the motion normal to the wall in the absence of external forces:

$$\frac{\partial f(z,t)}{\partial t} = \frac{\partial}{\partial z}\left(D_\perp(z) \frac{\partial f(z,t)}{\partial z} \right)$$

- Neuman boundary condition:

$$D_\perp(z) \frac{\partial f(z,t)}{\partial z}\bigg|_{z=a} = 0$$

Distribution of matched particles



- If the matched particles remained uniformly distributed, then $\langle z \rangle = z_C = (Z+a)/2$ for a window of extent $a \le z \le Z$ with particle radius *a.*
- Let *P(z)* be the probability density function (PDF) of distances from the wall that are sampled by matched particles. Then

$$P(z) = K \int_0^{\Delta t} f(z,t) f(z, \Delta t - t) dt$$

- *K* normalization constant, Δt measurement time.

Dimensional Analysis

- Three dimensionless variables:

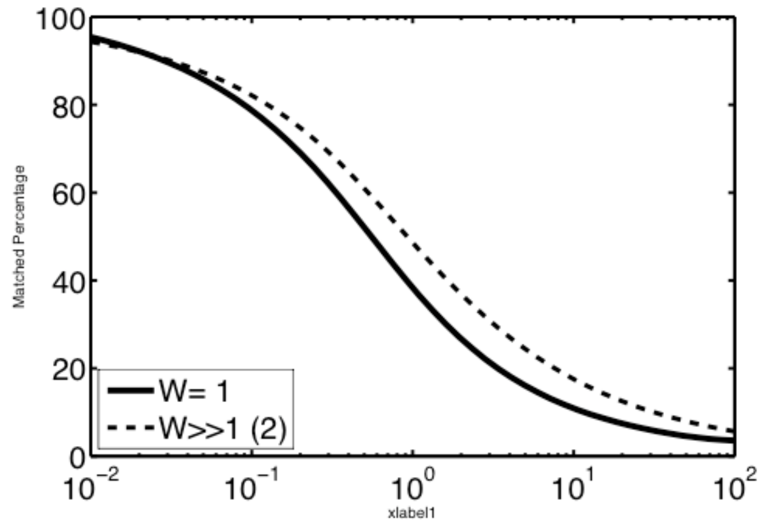
$$\zeta = \frac{z-a}{Z-a} \qquad W = \frac{Z-a}{a} \qquad \Omega = \frac{D_\infty \Delta t}{Z^2}$$

  Dimensionless edge-wall distance, window width and time.
- Natural choice as the characteristic length: *a*. *Z-a* is natural here, because we are interested in the role of diffusion over the length scale of the imaged region.
- Let *P*(ζ,Ω,W) be the PDF of dimensionless edge-wall distances sampled by matched particles. The imaged region becomes the interval [0,1].
- New dimensionless time:

$$\Omega_* = \frac{D_\infty \Delta t}{(Z+K_0 a)^2} \quad \text{with } K_0 = 0.8$$

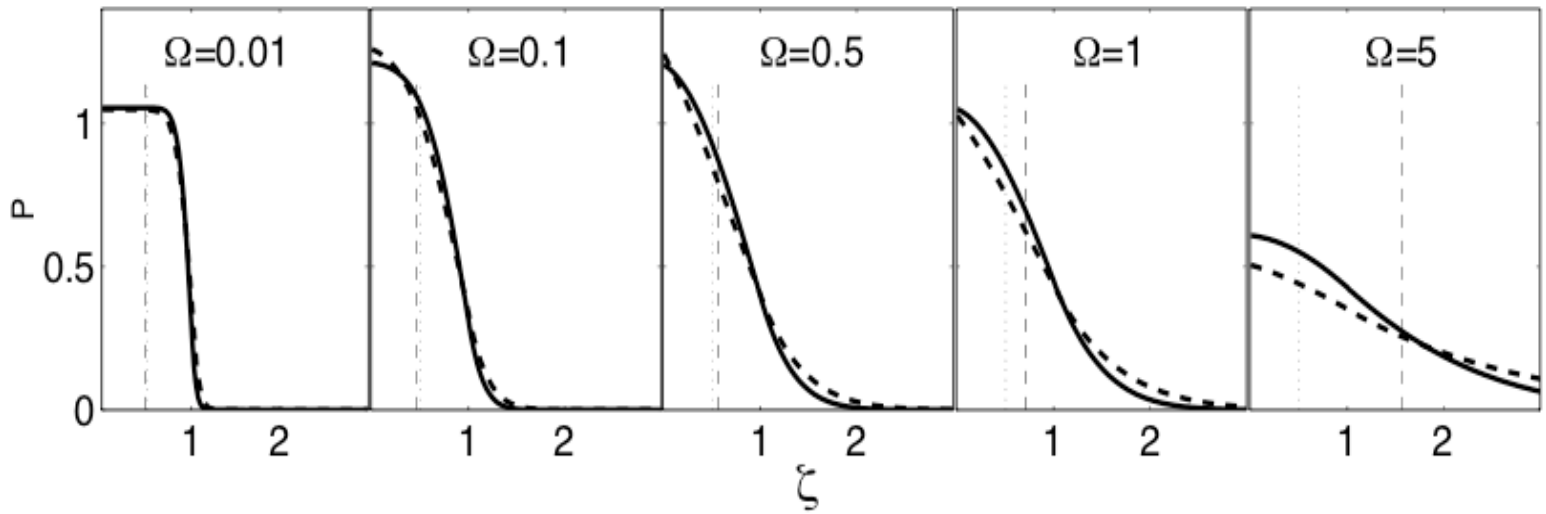
  to minimize the spread in $\langle z \rangle / z_C$ for Ω>1.

Result I



- Most tracer-based velocimetry technique assume <ζ>=0.5.
- When many particles drop-out of the window after the first exposure time, <ζ> shifts.

Result II



- The shift of <ζ> to the left for small Ω and infinitely to the right for larger Ω is independent of the window size *W*. The shifts in both direction are due to diffusion in an asymmetric environment.

Modeling

- In the limit when *W*»1, the window is so large that hindered diffusion is negligible:

$$\langle \zeta \rangle_\infty = \frac{1}{\Omega} \int_0^{\Omega} d\Omega' \int_0^{\infty} d\zeta \, \zeta f_\infty(\zeta, \Omega') f_\infty(\zeta, \Omega - \Omega') \ (1),$$

with

$$f_\infty(\zeta, \Omega) = \frac{1}{2F_\infty} \left(erf\left[\frac{\zeta + 1}{2\sqrt{\Omega}} \right] - erf\left[\frac{\zeta - 1}{2\sqrt{\Omega}} \right] \right)$$

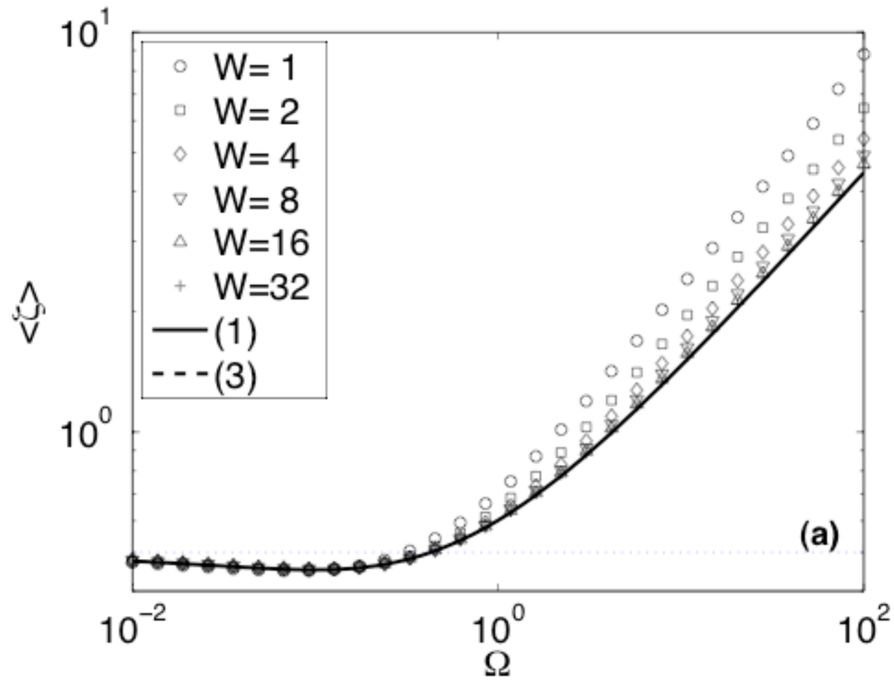
and

$$F_\infty = erf\left[\frac{1}{\sqrt{\Omega}} \right] + \left(e^{-1/\Omega} - 1 \right) \sqrt{\frac{\Omega}{\pi}} \ (2).$$

- The limiting curve $<\zeta>_\infty$ can be empirically approximated

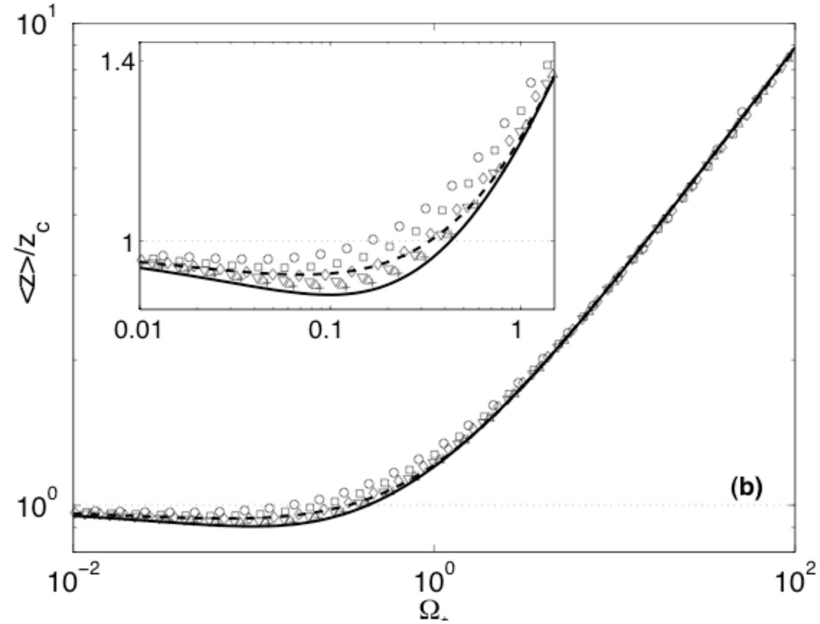
$$\frac{<z>}{z_C} \approx F(\Omega_*) = A + (1 - A)\exp(-B\sqrt{\Omega_*}) + C\sqrt{\Omega_*} \text{ with } A = 0.21,\ B = 1.72,\ C = 0.86 \ (3).$$

Modeling result I



- $<\zeta>$ as a function of Ω for different W and in the $W \gg 1$ limit

Modeling result II



- The inset shows the largest relative error appears when Ω_*~0.2. The relative error is within ~7% for *W*=1 and 5% for *W*≥2.
- The greatest scatter of the data while using Ω_* occurs for Ω_*<1, but in that case the *W*»1 limiting curve (dashed line) is a good approximation.

Conclusion

- The single-wall analysis is only valid when other geometric effects are relatively unimportant, for example when $Z \ll h$, the flow half-dimension, and $\Delta t \ll T_{CR}$, the time required for a tracer to diffuse across distance of $\sim O(0.1h)$.
- Brownian diffusion effects may possibly affect the measurements and lead to larger inferred slip length as reported by Lumma *et al.* for particle tracers.
- Brownian diffusion-induced bias should not be ignored in PIV measurements using quantum dots as tracers (see Pouya *et al.*).
- In the $W \gg 1$ limit it might be necessary to also consider the effect of a second wall.